Handicapping under Uncertainty
in an All-Pay Auction

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Abstract

A fundamental result of contest theory is that evenly matched contests are fought most intensely, implying that a contest designer maximizes effort from each contestant by artificially boosting the chances of the underdog. Such “handicapping” is credited with making sports contests more exciting, improving efficiency in internal labor markets, increasing effort from students competing to enter college, and raising revenues in auctions. We reexamine the handicapping problem in a two-period contest where the only information available on player ability is performance in the first period. When a contest is perfectly discriminating (i.e. an all-pay auction), the player who exerts the most effort wins, but the weaker player will not participate with some probability, resulting in lower total effort. However, we find that in the two-period contest, handicapping the loser of the first period increases total effort for all ability differences. When the objective is to increase accuracy in identifying the better player, handicapping the winner is optimal.

1 Introduction

Should a contest designer handicap the lower or higher ability player? Do firms rally around an outstanding employee for promotion or shift resources to the less able employee? Should a professor evaluate a star student stringently or leniently relative to an under achiever? In

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request for proposals, do firms prefer to even the chances of participating contractors so as to get higher quality (or lower price) bids? Should a search engine rank a less relevant site higher or lower than a site with quality information? In a contest with asymmetric players, total effort tends to be lower than if the players were closer in ability. It follows that the weaker player, having little chance of winning, only exerts a small amount of effort. The stronger player, already knowing he has a good chance of winning, pulls back as well.

This idea that contests are fought less intensely when players are not evenly matched is well understood in the theoretical literature on contests (Hillman and Riley 1989; Baye, Kovenock, and de Vries 1996). An obvious strategy is evening the contest by giving an advantage to the weaker player and increasing his chances of winning (Lazear and Rosen 1981; Che and Gale 2003; Fu 2006; Liu et al. 2007; Tsoulahas and Knoeber 2007).\(^1\) Just as giving a weaker golf player a “handicap” can make a golf match more intense, helping the weaker player can make all players compete harder to win the contest.

A problem with this approach that is not addressed in the literature is the contest designer does not always know the abilities of the players and will base her assessment in part on performance in a previous contest. But this would seem to create a clear incentive problem. For instance, a common problem in management is ratcheting, where employees reduce their current effort to obtain more favorable incentives in the future (Weitzman 1980). If there are (at least) two contests and players anticipate that winning the first contest will hurt them in the second contest, they will have less incentive to win the first contest. Therefore, favoring the loser of the first contest would seem to increase effort in the second contest, but at the expense of reducing each players’ incentives to win the first contest. We investigate this incentive problem in a two-period model between two players of unequal ability. They compete in a winner-take-all perfectly discriminating contest (better known as an all-pay auction) in each period where the loser or winner of the first period contest is given an advantage, or handicap, in the second period contest.\(^2\)

In an all-pay auction, the expected payoff to the weaker player is zero, so any handicapping

\(^1\)In fact the gains from making the contest more even can be so strong that total effort can often increase if the strongest player is removed from contest with \(N > 2\) players (Baye et al., 1993).

\(^2\)We use the term handicap popularized in golf usage where it advantages lower ability players.
would have to more than offset the asymmetry to induce any effort change. A handicap in the second period only has an effect on the first period incentives for the stronger player. But by reducing the incentives of the stronger player, the net effect is to also make the first period contest more even, implying that in equilibrium both players exert more effort. We find that handicapping the loser of the first period increases total effort in the second period. However, we find the anticipation of handicapping in the second period also increases total effort in the first period. Overall, total effort is increased since there is no incentive loss due to handicapping. This result contrasts with Meyer (1992) who uses a Lazear-Rosen difference-form contest to model two-period competition by two equally capable employees for job promotion. She finds that favoring the loser of the first period has a positive first-order effect on effort in the second period, but has a negative second-order effect on effort in the first period. In contrast, in an all-pay auction, we find that handicapping the loser has a positive first-order effect on effort in the second period, as well as a positive second-order effect in the first period. This paradoxical result is similar to the result in Ridlon and Shin (2010) with a Tullock ratio-form contest and only for very large ability differences. For an all-pay auction, however, handicapping is optimal over the entire range of ability differences.

Separate from the question of the effort-maximizing handicap is the question of how to maximize the predictiveness, or efficiency, of the contest. (Meyer, 1991; Clark and Riis, 2000) For instance, if the top two performers compete for a client sales account, are we more likely to know which sales representative is better if the top performer is given more resources or not? Under the standard handicapping policy of rewarding the loser of the first period we find that the stronger player has a stronger incentive to reduce effort in the first period than does the weaker player. The outcome of the first period is therefore a noisier predictor of the better player, which in turn makes the outcome of the second period noisier also. This reduces the predictive power of the contest under the standard handicapping policy, while increasing the power under a reverse handicapping policy.

The use of handicapping to increase accuracy also complements Meyer’s (1991) result for a two-period contest in which the players efforts are taken to be constant. She finds that if the player who wins in the first period is biased (i.e., given an advantage), then the second period
is very informative as to who is the better player. In contrast, if the player who loses in the first period is given an advantage then there is little extra information about which player is better if the other player wins in the second period. Therefore, a reverse handicap leads to the most accurate prediction of which player is better. Our model shows that allowing for strategic effort choices does not undermine this result, but instead further increases the precision from a reverse handicapping policy.

Most contest theory is derived from the economics literature beginning with Tullock (1980) on rent-seeking and Lazear and Rosen (1981) for optimal labor contracts. Although considerable research has been done on asymmetric all-pay auctions (Baye et al., 1993; Hillman and Riley, 1989; Clark and Riis, 2000; Che and Gale, 2003; Polborn, 2006; Berman and Katona, 2011), our model is the first to employ dynamic handicapping under uncertainty with this mechanism.

2 Model

2.1 One Period All-Pay Auction

We first examine a one period auction between players \( i = B, G \) who exert effort \( x_i = b, g \), respectively. The cost of effort is non-recoverable, i.e., investment is a sunk cost. The winner receives a prize, \( v_B \) if \( B \) wins and \( v_G \) if \( G \) wins. The player who loses receives a prize equal to zero. We assume the contest designer cannot directly monitor effort and only observes who is the winner. Both players want to maximize their own payoffs and the contest is won by whichever player exerts the most effort. Specifically, \( B \) wins \( v_B \) if \( \sigma b > g \), and if \( \sigma b < g \), \( G \) wins \( v_G \), where the parameter, \( \sigma \in [0, 1] \), measures the relative ability of the players.\(^3\) In other words, the contest is a perfectly discriminating contest mechanism. We restrict player \( B \) as the weaker player when \( \sigma < 1 \) and both players are equal in ability when \( \sigma = 1 \) without loss of generality. This parameter represents the effectiveness of one’s ability in exerting effort in the contest. This asymmetry in abilities could arise for a variety of reasons such as having access to fewer productive resources, being burdened with extra administrative duties, or simply lacks

\(^3\)Singh and Wittman (1998) use this form to represent differences in productivity of effectively transforming resources into contest effort. Polborn (2006) also uses this form to characterize the defense advantage factor in defending the status quo.
“natural talent”. Generically, each player possess different technologies in converting resources into productive effort. With effort having unit cost, player $G$’s payoffs are

$$\pi_G = \begin{cases} -g & \text{if } g \leq \sigma b, \\ v_G - g & \text{if } g > \sigma b. \end{cases}$$  \hspace{1cm} (1)$$

The payoffs to $B$ are similar and omitted.\textsuperscript{4} Notice that there can be no pure-strategy equilibrium since the other player will always have an incentive to deviate. Suppose for instance player $G$ exerts effort $g = v_B$. This would guarantee a win since player $B$ would not want to exert effort greater than $v_B$. Indeed, $B$ would rather not exert any effort at all since he will lose with certainty. In that case, $G$ should only exert a small amount of effort $\varepsilon > 0$ in order to maximize his payoff. In turn, $B$ would then have an incentive to exert $b > \varepsilon$. And so on. Therefore, there is no pure-strategy equilibrium, but an equilibrium can still exist in mixed-strategies.

We draw on the results from Polborn (2006) for an all-pay auction where players have asymmetric valuations and asymmetric abilities.\textsuperscript{5} He finds that the player with the combined higher valuation and stronger ability will have positive expected utility and the player with the lower valuation will have zero expected utility, while both still exert positive effort. Denote $F_i$ as the cumulative distribution function (cdf) of effort for $i$. In that case, the probability that $\sigma b > g$ is $F_G$.\textsuperscript{6} Let $i$’s payoff function be

$$\pi_i = F_jv_i - x_i,$$  \hspace{1cm} (2)$$

Since payoffs and efforts are dependent on relative valuation and ability asymmetry, the following are the cdf’s under each scenario. If $\sigma v_B < v_G$, then

$$F_B = \begin{cases} 1 - \frac{\sigma v_B}{v_G} & \text{for } b = 0, \\ 1 - \frac{\sigma v_B}{v_G} \frac{\sigma b}{v_G} & \text{for } b \in (0, v_B], \\ 1 & \text{for } b > v_B, \end{cases}$$  \hspace{1cm} (3)$$

and

$$F_G = \begin{cases} \frac{g}{\sigma v_B} & \text{for } g \in [0, \sigma v_B], \\ 1 & \text{for } g > \sigma v_B. \end{cases}$$  \hspace{1cm} (4)$$

\textsuperscript{4}We ignore the case of a tie since, as we will show, the probability of it occuring is atomless.

\textsuperscript{5}Note that $\upsilon$ and $\sigma$ are equivalent to $\Delta V$ and $1/r$, respectively, in Polborn (2006).

\textsuperscript{6}We follow standard techniques from Hillman and Riley (1989), Baye, et al. (1996) and Polborn (2006) for determining the mixed strategy equilibrium.
Note that when \( \sigma v_B < v_G \) player \( B \) exerts zero effort with probability \( (1 - \sigma v_B/v_G) \) where as player \( G \) has no probability mass with zero effort. This probability increases as the relative difference between \( \sigma v_B \) and \( v_G \) increases. If \( \sigma v_B \geq v_G \), then

\[
F_B = \begin{cases} \frac{\sigma b}{v_G} & \text{for } b \in [0, v_G/\sigma], \\ 1 & \text{for } b > v_G/\sigma, \end{cases}
\]  

(5)

and

\[
F_G = \begin{cases} 1 - \frac{v_G}{\sigma v_B} & \text{for } g = 0, \\ 1 - \frac{v_G}{\sigma v_B} + \frac{g}{\sigma v_B} & \text{for } g \in (0, \sigma v_B], \\ 1 & \text{for } g > \sigma v_B. \end{cases}
\]  

(6)

The cdf’s are linear and the slope of the relatively weaker player decreases as the players become more asymmetric, resulting in greater probability of zero effort. Expected individual efforts are calculated as

\[
E(b) = \begin{cases} \frac{\sigma v_B v_B}{v_G^2} & \text{if } \sigma v_B < v_G, \\ \frac{v_G}{2\sigma} & \text{if } \sigma v_B \geq v_G, \end{cases}
\]  

(7)

\[
E(g) = \begin{cases} \frac{\sigma v_B}{\sigma^2} & \text{if } \sigma v_B < v_G, \\ \frac{v_G v_B}{2\sigma^2} & \text{if } \sigma v_B \geq v_G. \end{cases}
\]  

(8)

Based on the cdf’s and efforts, let \( p_G \) denote the equilibrium probability \( G \) wins the contest as

\[
p_G = \begin{cases} 1 - \sigma v_B/2v_G & \text{if } \sigma v_B < v_G, \\ v_G/2\sigma v_B & \text{if } \sigma v_B \geq v_G. \end{cases}
\]  

(9)

In other words, whichever player has the higher combined relative valuation and ability has the higher expected probability of winning the contest. Expected payoffs for players \( G \) and \( B \) are

\[
E(\pi_G) = \begin{cases} v_G - \sigma v_B & \text{if } \sigma v_B < v_G, \\ 0 & \text{if } \sigma v_B \geq v_G, \end{cases}
\]  

(10)

\[
E(\pi_B) = \begin{cases} 0 & \text{if } \sigma v_B < v_G, \\ v_B - v_G/\sigma & \text{if } \sigma v_B \geq v_G. \end{cases}
\]  

(11)

Consistent with all-pay auctions, the player with the higher valuation earns positive expected profits, while the player with the lower valuation earns zero expected profits. This, of course,
is moderated by asymmetric abilities. From Equations 7 and 8, total effort is highest when the players are symmetric both at $v_B = v_G$ and $\sigma = 1$. At this point, the prize is fully dissipated, that is, the expected effort is equal to the expected payoff. However, as asymmetry increases, total expected effort is diminished. The contest designer has an incentive to reduce the ability differences between the players when $\sigma v_B < v_G$ so as to increase total effort. Since most managers want to maximize effort, this can be advantageous not in the sense of more hours, but arises from the increased productivity and benefits from effort. For instance, more effort spent in preparation of a sales call can lead to not only an increase in probability of winning the account, but also in the expected value of the sale through negotiations.

Suppose that the designer can allocate a handicap $h$ to one of the players giving them an advantage in ability. For example, reducing administrative tasks, making resources more available, etc. to the weaker player. Another way of looking at it is increasing administrative tasks or making resources more scarce for the stronger player. The handicap has a multiplicative effect on effort as in Clark and Riis (2000), such that $B$ wins if $h \sigma b > g$. Obviously, the contest designer maximizes effort by setting

$$h = \begin{cases} 
\frac{v_G}{\sigma v_B} & \text{if } \sigma v_B < v_G, \\
\frac{\sigma v_G}{v_B} & \text{if } \sigma v_B \geq v_G.
\end{cases}$$

(12)

However, this policy is only possible when the contest designer has complete information on the identity of the weaker player. If she does not have complete information, there is a possibility the handicap is erroneously applied to the stronger player. We explore this problem in the next section.

### 2.2 Uncertainty and Handicapping

We now assume the contest designer knows $\sigma$, but receives a noisy signal of which player is truly the weaker player. The signal is such that with some probability, $p$, the contest designer can correctly identify the weaker player and apply the handicap. There is also a probability $(1 - p)$ the handicap is erroneously allocated to the unintended player. She adjusts the handicap when taking this uncertainty into account.

We restrict the value of winning the contest equal to $v_B = v_G = v$ without loss of generality.
In this way, the condition $\sigma v_B \leq v_G$ is satisfied when $\sigma < h < 1/\sigma$.\footnote{When $h$ is not in this range, the efforts change according to Equations (7) and (8). See appendix for a full analysis of $h$.} With probability $p$ the contest designer correctly identifies the weaker player and allocates a handicap such that efforts simplify to

$$b_L = g_W = \frac{h\sigma}{2}v,$$

where $b_L$ is the effort of player $B$ correctly identified as the losing player, and $g_W$ is the effort of player $G$ correctly identified as the winning player. However, with probability $(1 - p)$ the contest designer incorrectly identifies the weaker player, resulting in an handicap allocation where efforts are

$$b_W = g_L = \frac{\sigma}{2h}v. \quad (14)$$

The contest designer maximizes expected effort

$$E^{b+g} = p (h\sigma v) + (1 - p) \frac{\sigma}{h}v$$

with respect to $h$ depending on $p$ subject to the valuation inequalities. These inequalities simplify to $h\sigma \leq 1$ and $\sigma/h \leq 1$ when valuations are equal.

**Proposition 1** In the one period case with a noisy signal, the contest designer always maximizes effort at $h = \frac{1}{\sigma}$ for all $p$ and $\sigma$.

**Proof.** If the contest designer over-handicaps, the inequality changes to where the weaker player’s ability is handicapped in excess of the stronger player’s ability, resulting in lower total efforts. It is easy to see given when the contest designer has full information, i.e., $p = 1$, the optimal handicap is $h = 1/\sigma$ and the prize is fully dissipated. If, however, she has useless information, i.e., $p = 1/2$, the optimal handicap is still $h = 1/\sigma$. The reason for this is that with probability $\frac{1}{2}$ the contest is fully dissipated as in the full information case, but with probability $\frac{1}{2}$ total effort is less than with no handicapping, and always positive. This is easily verified by the inequality $2b_L - \sigma > \sigma - 2b_W$ for all $\sigma$ and $1 \leq h \leq 1/\sigma$. Hence, expected effort with random handicapping dominates all other handicapping policies in the one period contest only when $p = 1/2$. We find that even for $p < 1$, the contest designer still handicaps the “suspected loser” with $h = \frac{1}{\sigma}$, since $\frac{\partial E^{b+g}}{\partial h} > 1$ when $h = 1$.
3 Two Period All-Pay Auction

3.1 Maximizing Effort

As stated in the previous section, the signal of ability could come from different aspects of the work environment, but then we need to put some structure on how the signal maps onto ability. Just like in sports, a good predictor of players' true abilities is past performance. For this reason, we introduce a two period contest. Looking at this from a two period perspective, it is easy to see the result of any handicapping policy will affect first period efforts. In the first period, the players compete to win the prize just as described in the earlier sections. A win in the first period signals the player is truly the stronger player with some probability. The structure on the expected probability of winning is dependent on the mixed strategy equilibrium calculated in the previous section. Based on this information, the contest designer can choose a handicap that will increase total effort not only in the second period contest, but for both contests combined. The contest designer credibly commits to a handicap prior to the first period contest. This handicap will be assigned to the loser of the first period contest, but prior to the second period contest. A handicap of \( h > 1 \) benefits the loser while a handicap of \( h < 1 \) favors the winner and puts the loser at a disadvantage.

Let \( v_B = v_G = v \) be the value of winning the contest in each period for simplicity. In this way, the only \( \sigma \) and \( h \) will determine the probability of winning, total effort, and expected payoffs in equilibrium. For instance, if \( h > 1 \), then the players have an incentive to lose the first period contest by reducing there efforts in order to gain the advantage in the second period. However, the loss in the first period may be greater than the gain in the second period depending on the value of \( h \). If \( h < 1 \), or a reverse handicap, then the players have a stronger incentive to win the first period contest so as to have the advantage in the second period. We will evaluate when and if these behaviors occur, and how it impacts total effort.

The size of \( h \) affects the relationship of valuations. There are then two outcomes (win or lose) and two conditions (\( B \)'s valuation is relatively greater than or less than \( G \)'s valuation depending on \( \sigma \) and \( h \)) for each player. For example, if \( h \) is very large, then \( h\sigma v_B \geq v_G \), but since \( v_B = v_G = v \), it reduces to \( h\sigma \geq 1 \). In a sequential game, we use backward induction to
find the sub-game perfect Nash equilibrium. We first determine the second period equilibrium efforts based on $\sigma$ and $h$. From this, we can determine the optimal action in the first period contest. There are two possible scenarios in the second period. First, if $G$ wins the first period auction, then his expected payoffs for the second period auction are

$$E\left(\pi_{G,W}(\sigma, h)\right) = \begin{cases} 
(1 - h\sigma) v & \text{if } h\sigma < 1, \\
0 & \text{if } h\sigma \geq 1,
\end{cases}$$

(15)

where the subscript $G, W$ reads $G$ wins the first period auction. Player $B$’s expected payoffs for the second period auction are

$$E(\pi_{B,L}(\sigma, h)) = \begin{cases} 
0 & \text{if } h\sigma < 1, \\
(1 - 1/(h\sigma)) v & \text{if } h\sigma \geq 1.
\end{cases}$$

(16)

where the subscript $B, L$ reads $B$ loses the first period auction. The second scenario is if player $B$ wins the first period auction, then expected payoffs to $G$ and $B$ are

$$E(\pi_{G,L}(\sigma, h)) = \begin{cases} 
(1 - \sigma/h) v & \text{if } \sigma/h < 1, \\
0 & \text{if } \sigma/h \geq 1,
\end{cases}$$

(17)

$$E(\pi_{B,W}(\sigma, h)) = \begin{cases} 
0 & \text{if } \sigma/h < 1, \\
(1 - h/\sigma) v & \text{if } \sigma/h \geq 1.
\end{cases}$$

(18)

Depending on the handicap, the players may have an incentive to lose in first period in the form of decreased implicit valuations in the first period. The impact of the handicap in the second period on first period valuation is based on the difference of payoffs in the second period given they either won or lost in the first period. The expected payoff to player $i$ for both periods is

$$E(\pi_i(\sigma, h)) = p_i v - x_i + p_i (\pi_{i,W}) + (1 - p_i) (\pi_{i,L}).$$

(19)

We now denote $\omega_i$ as player $i$’s implicit valuation in the first period, including the implicit gain in the second period from winning the first period. Player $i$’s first period implicit valuation is written as

$$\omega_i = v + \pi_{i,W} - \pi_{i,L},$$

(20)
Figure 1: Equilibrium Effort Distributions

First Period

A) Without Handicap

B) With Handicap

where \( \pi_{iW} - \pi_{iL} \) is the “bonus” of winning in the first period from second period payoffs. By internalizing the future benefit of winning (or losing) the first period contest, players’ efforts are endogenous to the handicapping policy of the contest designer. Note that \( \frac{\partial}{\partial h} \pi_G < 0 \) and player B’s payoff function is always zero under case 1. Since the contest is more even, both players will expend more effort. This implies that when the players’ valuations become closer they work harder while earning less profit, benefiting the contest designer.

Proposition 2 Effort under a handicapping policy is higher than under a reverse handicapping policy.

Proof is in Appendix. We prove that the optimal handicap is greater than one, but less than \( 1/\sigma \). Specifically, the optimal handicap that maximizes effort is given as

\[
h^* = \frac{(1 - \sigma) + \left(1 - 2\sigma + 5\sigma^2\right)^{\frac{1}{2}}}{2\sigma},
\]

which is greater than one. Player G’s valuation in the first period is strictly greater than player B’s valuation. When \( h = h^* \) the implicit first period valuations are \( \sigma \omega_B = \omega_G \). A handicap of \( h^* > 1 \) reduces player G’s valuation while not affecting player B’s valuation. However, this makes the auction more even in the first period, resulting in increased total effort. The graphs in Figure (1) are the cdf’s of the mixed strategies used by the players in each period. They illustrate the impact of a handicapping policy on players’ effort behavior. Notice in Figure
(1A) that when no handicap is announced \( (h = 1) \), \( (1 - \sigma) \) percent of time \( B \) does not exert any effort. Figure (1B) is when the optimal handicap is announced. Notice that the optimal handicap reduces the probability of player \( B \) exerting no effort to zero, shifting the function down. Although, \( G \)'s implicit value in the first period is reduced by the handicap, his optimal response remains unchanged. This is counterintuitive since we would expect player \( G \) to lessen his efforts in response to a lowered incentive to win in the first period. However, \( B \)'s expected probability of winning is now equal to player \( G \) since both have identical incentives to win the first period and his cdf changes to \( F_B = \sigma F_G \). So although \( G \)'s value is reduced, he still retains his ability advantage and \( B \) increases his expected effort.

Figure (2) illustrates the findings of Proposition 2. The optimal handicap is a decreasing function of ability differences. When the players are very different in abilities, the contest designer heavily handicaps the loser in the second period. As the players become more similar in abilities, the incentive effect diminishes, and the designer still favors the loser, but to a lesser degree.

Total effort expended in both periods is a function of ability differences and the handicap. Recall that as players become more even in abilities, total effort increases. In a deterministic setting, this leads to full dissipation of the prize, where total effort equals the value of the
prize. Without handicapping, total effort tends to zero as players become sufficiently different in abilities. Introducing a handicapping policy restores much of the effort lost due to asymmetry. As can be seen from Figure (2), there is a significant benefit to handicapping the loser of the first period contest in terms of gained effort. This benefit remains positive, but diminishes as the difference in ability becomes smaller. That is, when players are identical in abilities, the prize is fully dissipated and total effort is maximized.

3.2 Maximizing Predictiveness

While maximizing total effort yields higher productive results, ensuring the contest mechanism allocates the prize to the player who values the prize the most, or is the most efficient in effort may be preferred by the contest designer. For example, accurately promoting the higher ability player to a function sensitive to effort effectiveness has a long-term benefit greater than maximizing effort in a low productive function. If the abilities of the players are unknown by the contest designer, however, then the outcome of the contest acts as a signal of abilities. In this section, the objective of the contest designer is to increase the precision, or efficiency, in identifying the higher ability player. The players’ objectives of maximizing payoffs in both periods remain unchanged. For instance, a sales manager wants to promote the most effective salesperson to a more profitable account, or a firm wants the agency with the most innovative ideas for its advertising campaign. By implementing a contest, the contest designer can verify the identity of the better player with a certain degree of accuracy by observing the winner. In the previous section, we showed that since players draw efforts from a equilibrium distribution functions, then success remains probabilistic. We examine the efficiency of the two-period contest model by analyzing the effect of $h$ on the higher ability player’s probability of winning each contest. Recall that when the objective is to maximize total effort, then handicapping (i.e., $h > 1$) the loser of the first period contest strictly dominates reverse handicapping for all ability differences. Both players increase efforts since the contest is more even, and increasing the probability of a win by the weaker player. The probability of $G$ winning the first period contest comes from Equation (9) and substituting (20) for $v$. As such, the effect of $h$ on relative first period valuations, as well as the relative ability gap, is critical to the probability of success. Furthermore, there are
two decision rules the contest designer can use to identify the higher ability player. She can use the winner of the first period (Rule 1), or the winner of the second period (Rule 2) as a signal of ability.\footnote{We assume the contest designer privately chooses which rule to use so as not to change the structure or the information of the contest.} Rule 1 is the most intuitive since $h$ has a first period incentive effect. Using Rule 2, on the other hand, is still conditional on the first period contest since $h$ may or may not be assigned to the truly stronger player. Taking into account relative first period valuations, rules, and strategic effects, the following proposition describes the effect of $h$ on efficiency.

**Proposition 3** A handicap is always less predictive of ability than a reverse handicap.

Proof of Proposition 3 is in the Appendix. We find that the optimal handicapping policy for predicting the high ability player is to reverse handicap, specifically $h = \sigma$. A reverse handicap, i.e., $h < 1$, has two effects on the probability of success. It increases the ability differences between the players in the second period, and it increases the value of winning to the higher ability player in the first period. Indeed, while player $B$’s first period valuation remains unaffected by a reverse handicap, the marginal effect on player $G$’s first period valuation is $\sigma (1 + h^2) / h^2$. 

Figure 3: Increase in Accuracy from Reverse Handicapping as a Function of Player Asymmetry
Figure (3) shows the improvement in accuracy versus no handicapping under each rule in absolute terms. While the graph suggests that Rule 1 has a larger impact, reverse handicapping strictly dominates handicapping under both rules. In other words, a reverse handicap leads to a higher efficiency even when abilities are unknown.

3.3 Commitment

Recall that in the prior two sections, the contest designer committed to a handicapping policy prior to the beginning of the first period contest. In some cases, however, no such mechanism for credibly committing exists. In this case, the contest designer cannot strategically modify behavior by announcing a handicapping policy before the contest. But this may prove difficult to the designer, leading to uncertainty and speculation by the players about the potential handicapping policy prior to the second period contest. In the absence of commitment, the contests designer still only has information on the identity of the weaker player, but has an incentive to even the contest in the second period by handicapping the loser.

**Proposition 4** A handicapping policy without commitment is lower than a handicapping policy with commitment. Total effort without commitment is lower than total effort with commitment.

Proof is in appendix. Since the optimal handicap with commitment also favors the loser, the inability to commit does not change the magnitude of the policy. As such, the handicap with commitment is greater than without commitment. As a result, since $h$ is less than $h^*$ total efforts are lower than with commitment. More importantly, however, total effort is still greater than not handicapping at all.

4 Conclusion

This paper has demonstrated an application of the all-pay auction in a sales contest setting. Most of the literature on sales contests and all-pay auctions have assumed either homogenous players or the sales manager knows the abilities of each player. This paper simultaneously examines both issues. First, players exert less effort when a contest is unevenly matched. Through the use of a dynamic handicapping policy in a multi-period contest, incentives are implicitly made
more even, inducing both players to increase their total contest efforts. This paper defines the optimal handicapping policy for the contest designer dependent on the relative ability gap between players. The second problem is the efficiency in identifying the higher ability player. When abilities are unknown, identifying the stronger player with greater accuracy is a common objective of the contest designer. Through a reverse handicapping policy, which favors the winner of the first period contest, the contest designer increases the incentive of winning the first period inducing the stronger player to exert more effort in the first period. This increases the likelihood of being identified as the stronger player in either period. This paper has contributed to the theoretical research on sales contests and outlines specific handicapping policies for the contest designer in an all-pay auction context.

Although handicapping the weaker player increases total effort, the extra advantage may seem unfair and cause disenchantment of the stronger player in an employee setting. Furthermore, if identification of the stronger player leads to a higher individual payoff in the future, players would internalize this incentive, muting the effect of a reverse handicap. Another limitation to the model is our focus on the two-player case whereas other research examine a much larger pool of participants. It is reasonable to assume, however, that the contest designer may only have to distinguish between two players, or perhaps it is not practical to allow more than two players to compete for the same prize. Its application to the asymmetric multi-player case as in Baye, Kovenock, and de Vries (1993) would broaden its applications. The current paper uses an all-pay auction mechanism which is equivalent to the ratio-form contest when $R = 1$. While Ridlon and Shin (2010) solve the equilibrium handicapping policy for $R = 1$, research on the effect of intermediate values of $R$ on equilibrium handicapping policies would particularly interesting as to when the contest designer switches from exclusively using a handicapping strategy to a reverse handicapping strategy depending on asymmetry.

5 Appendix

Proof of Proposition 2:

Let $\omega^c_i$ be the valuation of the first period to player $i$ under case $c$, and $E^c_i$ be the combined
efforts of both players in period \( t \) in case \( c \). Note that the players' valuations depend on the impact of \( \sigma \) and \( h \) as indicated in the following three cases for each player.\(^9\)

\[
\omega_G^c = v + \begin{cases} 
((1 - h\sigma) - (1 - \sigma/h))v & \text{if } h\sigma < 1, \ \text{and } \sigma/h < 1, \ \ c = 1 \\
((1 - h\sigma) - 0)v & \text{if } h\sigma < 1, \ \text{and } \sigma/h \geq 1, \ \ c = 2 \\
(0 - (1 - \sigma/h))v & \text{if } h\sigma \geq 1, \ \text{and } \sigma/h < 1, \ \ c = 3 
\end{cases}
\]

\[
\omega_B^c = v + \begin{cases} 
(0 - 0) & \text{if } h\sigma < 1, \ \text{and } \sigma/h < 1, \ \ c = 1 \\
((1 - h/\sigma) - 0)v & \text{if } h\sigma < 1, \ \text{and } h/\sigma \geq 1, \ \ c = 2 \\
(0 - (1 - 1/(h\sigma)))v & \text{if } h\sigma \geq 1\text{and } \sigma/h < 1, \ \ c = 3 
\end{cases}
\]

These first period valuations vary in that some are invariant to \( h \) or \( \sigma \), some are decreasing, and some are increasing. Recall that players mix their efforts over a range based on their valuations.

First case \((c = 1)\): \( \sigma < h < 1/\sigma \)

Under the first case, \( h\sigma < 1 \) and \( \sigma/h < 1 \), or simply \( \sigma < h < 1/\sigma \), the valuations to the players are \( \omega_G^1 = v(1 + (1 - h\sigma) - (1 - \sigma/h)) \) and \( \omega_B^1 = v \). Note that \( \frac{\partial}{\partial h}\omega_G^1 < 0 \) and \( \frac{\partial}{\partial h}\omega_B^1 = 0 \), so a handicap will reduce only player \( G \)'s value of winning the first round. Total effort in first period for case 1, if \( \sigma\omega_B^1 > \omega_G^1 \), is \( E_1 = \frac{1}{2\sigma}(\omega_G^1 + (\omega_G^1)^2/\omega_B^1) \) and \( \frac{\partial}{\partial h}\frac{1}{2\sigma}(\omega_G^1 + (\omega_G^1)^2/\omega_B^1) < 0 \). Total effort in first period, if \( \sigma\omega_B^1 < \omega_G^1 \), is \( E_1 = \frac{\sigma}{2} (\omega_B^1 + (\omega_B^1)^2/\omega_G^1) \) and \( \frac{\partial}{\partial h}\frac{\sigma}{2} (\omega_B^1 + (\omega_B^1)^2/\omega_G^1) > 0 \). Note that effort is increasing in \( h \) when \( \sigma\omega_B^1 < \omega_G^1 \) and decreasing in \( h \) when \( \sigma\omega_B^1 > \omega_G^1 \). However, when \( h \) increases to \( h^* = \left(1 - \sigma + (1 - 2\sigma + 5\sigma^2)^{1/2}\right)/2\sigma < \frac{1}{\sigma}, \ \ \sigma\omega_B^1 = \omega_G^1 \). At this point, total effort is maximized for both conditions. Total effort in second period is

\[
E_2 = p_G (g_W + b_L) + (1 - p_G) (g_L + b_W).
\]

Combined efforts for both periods simplify to

\[
(E_1^1 + E_2^1) = E^1 = \sigma (h + 1) v, \text{ subject to } \sigma\omega_B^1 \leq \omega_G^1.
\]

Substituting \( h \) for

\[
h^* = \left(1 - \sigma + (1 - 2\sigma + 5\sigma^2)^{1/2}\right)/2\sigma,
\]

we find that

\[
E^1 (h = h^*) > E^1 (h = 1),
\]

\(^9\)A fourth case exists where \( h\sigma \leq 1 \) and \( \sigma/h \geq 1 \), but the conditions are contradictory and therefore omitted.
for all $\sigma$, so handicapping clearly increases total effort in case 1. This is due to the handicap reducing the value of winning in the first period only for player $G$. This makes the players’ valuations in the first period closer together, leading to the contest being more evenly matched, and increasing total first period effort.

Second case ($c = 2$): $h < \sigma$

In the second case, $h\sigma < 1$, and $\sigma/h \geq 1$, or simply $h < \sigma$, the players’ valuations are $\omega^2_G = (2 - h\sigma)v$ and $\omega^2_B = (2 - h\sigma)^2v$, and only condition $\sigma\omega^2_B < \omega^2_G$ holds. It follows that $\frac{\partial}{\partial h}\omega^2_B < \frac{\partial}{\partial h}\omega^2_G < 0$, meaning that player $B$’s valuation decreases faster than player $G$’s valuation as $h$ increases. Therefore, as $h$ decreases, their valuations become closer together and more effort is exerted. Total effort in first period for case 2 is

$$E^2_1 = \frac{\sigma}{2} \left( \omega^2_B + (\omega^2_B)^2 / \omega^2_G \right),$$

and $\frac{\partial}{\partial h} E^2_1 < 0$. Total effort in second period is

$$E^2_2 = p_G (g_W + b_L) + (1 - p_G) (g_L + b_W).$$

and $\frac{\partial}{\partial h} E^2_2 > 0$. Combined efforts for both periods in case 2 simplifies to

$$(E^2_1 + E^2_2) = E^2 = (h\sigma + 2\sigma - h)v, \text{ if } h < \sigma.$$ (30)

This is maximized at $h = 0$, and is identical to $h = 1$ in case 1. However, this is less efficient than $h^*$ in case 1 where

$$E^2 (h = 0) < E^1 (h = h^*).$$

(31)

Third case ($c = 3$): $h > 1/\sigma$

The third case is where $h\sigma \geq 1$ and $\sigma/h < 1$, or simply $h > 1/\sigma$, the players’ valuations are $\omega^3_G = \sigma/h$ and $\omega^3_B = 1/(h\sigma)$, and only condition $\sigma\omega^3_B > \omega^3_G$ holds. Since $\frac{\partial}{\partial h}\omega^3_B > \frac{\partial}{\partial h}\omega^3_G > 0$, then the difference in the players’ valuations is increasing in $h$, making the contests less competitive, driving total effort in the first period down. Indeed, total effort in first period is

$$E^3_1 = \frac{1}{2\sigma} \left( \omega^3_G + (\omega^3_G)^2 / \omega^3_B \right),$$

and $\frac{\partial}{\partial h} E^3_1 < 0$. Total second period effort is given as

$$E^2_2 = p_G (g_W + b_L) + (1 - p_G) (g_L + b_W),$$

(33)
and $\frac{d}{dh} E_2^3 < 0$. Combined efforts for both periods in case 3 simplifies to:

\[
(E_1^3 + E_2^3) = E^3 = \frac{(\sigma + 1)}{h} v, \text{ if } h > 1/\sigma,
\]

Total effort is maximized at $h = 1/\sigma$, but

\[
E^3 (h = 1/\sigma) < E^1 (h = h^*), \quad (35)
\]

so clearly $h = h^*$ is the optimal handicapping policy such that it maximizes total effort. □

**Proof of Proposition 3:**

From Polborn (2006) we find the conditional probabilities of player $i$ winning the second round given player $j$ has won the first round ($\Pr(i \mid j)$) for each case is

\[
\begin{align*}
\Pr(G \mid G) : \quad & \left\{ \begin{array}{ll}
(1 - \frac{\sigma \omega_B}{2\omega_G}) (1 - \frac{ha}{2}) & \text{if } \sigma \omega_B < \omega_G \text{ and } h \sigma < 1 \\
(1 - \frac{\sigma \omega_B}{2\omega_G}) \frac{1}{2h\sigma} & \text{if } \sigma \omega_B < \omega_G \text{ and } h \sigma \geq 1 \\
\frac{\omega_G}{2\sigma \omega_B} (1 - \frac{ha}{2}) & \text{if } \sigma \omega_B \geq \omega_G \text{ and } h \sigma < 1 \\
\frac{\omega_G}{2\sigma \omega_B} \frac{1}{2h\sigma} & \text{if } \sigma \omega_B \geq \omega_G \text{ and } h \sigma \geq 1 
\end{array} \right., \\
\Pr(G \mid B) : \quad & \left\{ \begin{array}{ll}
(1 - \frac{\sigma \omega_B}{2\omega_G}) \frac{ha}{2} & \text{if } \sigma \omega_B < \omega_G \text{ and } h \sigma < 1 \\
(1 - \frac{\sigma \omega_B}{2\omega_G}) (1 - \frac{1}{2h\sigma}) & \text{if } \sigma \omega_B < \omega_G \text{ and } h \sigma \geq 1 \\
\frac{\omega_G}{2\sigma \omega_B} \frac{ha}{2} & \text{if } \sigma \omega_B \geq \omega_G \text{ and } h \sigma < 1 \\
\frac{\omega_G}{2\sigma \omega_B} (1 - \frac{1}{2h\sigma}) & \text{if } \sigma \omega_B \geq \omega_G \text{ and } h \sigma \geq 1 
\end{array} \right., \\
\Pr(B \mid G) : \quad & \left\{ \begin{array}{ll}
\frac{\sigma \omega_B}{2\omega_G} (1 - \frac{\sigma}{h}) & \text{if } \sigma \omega_B < \omega_G \text{ and } \sigma/h < 1 \\
\frac{\sigma \omega_B}{2\omega_G} \frac{1}{2h} & \text{if } \sigma \omega_B < \omega_G \text{ and } \sigma/h \geq 1 \\
(1 - \frac{\omega_G}{2\sigma \omega_B}) (1 - \frac{\sigma}{h}) & \text{if } \sigma \omega_B \geq \omega_G \text{ and } \sigma/h < 1 \\
(1 - \frac{\omega_G}{2\sigma \omega_B}) \frac{1}{2h} & \text{if } \sigma \omega_B \geq \omega_G \text{ and } \sigma/h \geq 1 
\end{array} \right., \\
\Pr(B \mid B) : \quad & \left\{ \begin{array}{ll}
\frac{\sigma \omega_B}{2\omega_G} \frac{\sigma}{h} & \text{if } \sigma \omega_B < \omega_G \text{ and } \sigma/h < 1 \\
\frac{\sigma \omega_B}{2\omega_G} (1 - \frac{1}{2h}) & \text{if } \sigma \omega_B < \omega_G \text{ and } \sigma/h \geq 1 \\
(1 - \frac{\omega_G}{2\sigma \omega_B}) \frac{\sigma}{h} & \text{if } \sigma \omega_B \geq \omega_G \text{ and } \sigma/h < 1 \\
(1 - \frac{\omega_G}{2\sigma \omega_B}) (1 - \frac{1}{2h}) & \text{if } \sigma \omega_B \geq \omega_G \text{ and } \sigma/h \geq 1 
\end{array} \right., \quad (36, 37, 38, 39)
\end{align*}
\]

The proof of the proposition follows by evaluating the probability of choosing player $G$ for different cases and different decision rules. The designer chooses from two different rules in
determining who the higher ability player is based on performance in the two period contest. The first rule is to choose the winner of the first period (Rule 1). The expected probability that player $G$ is the winner of the first round is calculated as

$$P_G^{c,1} = \frac{(\Pr(G, G) + \Pr(B, B)) \Pr(G, G)}{(\Pr(G, G) + \Pr(B, B))} + \frac{(\Pr(G, B) + \Pr(B, G)) \Pr(G, B)}{(\Pr(G, B) + \Pr(B, G))},$$

(40)

where $c$ is the case and 1 is the first rule. The second rule is to choose the winner of the second period (Rule 2). The expected probability that player $G$ is the winner of the second round is calculated as

$$P_G^{c,2} = \frac{(\Pr(G, G) + \Pr(B, B)) \Pr(G, G)}{(\Pr(G, G) + \Pr(B, B))} + \frac{(\Pr(G, B) + \Pr(B, G)) \Pr(B, G)}{(\Pr(G, B) + \Pr(B, G))}.$$  

(41)

Using $h = 1$ as the base case (which is restricted to the constraints in case 1), the probability that player $G$ is correctly identified as the high ability player using Rule 1, which is identical to using Rule 2, is

$$p_{\text{base}} = 1 - \frac{1}{2\sigma}.$$  

(42)

**Case 1:** $h\sigma < 1$ and $\sigma/h < 1$, and $\omega^1_G = 1 + \sigma/h - h\sigma$ and $\omega^1_B = 1$, implying $\sigma \omega^1_B < \omega^1_G$.

Case 1, Rule 1: the probability that identify player $G$ is simply

$$p^{1,1} = \left(1 - \frac{\sigma \omega^1_B}{2\omega^1_G}\right),$$

(43)

which is greater than $p_{\text{base}}$ when $h < 1$, and less than $p_{\text{base}}$ when $1 < h < \frac{1}{\sigma}$.

Case 1, Rule 2: The probability that identify player $G$ is

$$p^{1,2} = \left(1 - \frac{\sigma \omega^1_B}{2\omega^1_G}\right) \left(1 - \frac{h\sigma}{2}\right) + \frac{\sigma \omega^1_B}{2\omega^1_G} \left(1 - \frac{\sigma/h}{2}\right),$$

(44)

which is greater than $p_{\text{base}}$ when $h < 1$, and less than $p_{\text{base}}$ when $1 < h < \frac{1}{\sigma}$. Therefore, $h < 1$ is a better predictor of identifying player $G$. In fact, predictability is maximized when $h = \sigma$.

**Case 2:** $h\sigma < 1$ and $\sigma/h > 1$, $\omega^2_G = 2 - h\sigma$ and $\omega^2_B = 2 - h/\sigma$, implying $\sigma \omega^2_B < \omega^2_G$.

Case 2, Rule 1: The probability that identify player $G$, given as

$$p^{2,1} = \left(1 - \frac{\sigma \omega^2_B}{2\omega^2_G}\right),$$

(45)

10 Under case 1, for $h < \frac{(1-\sigma + \sqrt{2\sigma + 5\sigma^2 + 1})}{2\sigma} < 1/\sigma$ condition switches to $\sigma \omega^b_G > \omega^b_B$. However, this yields probability less than when $h = 1$.  

20
is greater than $p^{\text{base}}$ for all $h < \sigma$.

Case 2, Rule 2: The probability that identify player $G$, given as

$$p^{2.2} = \left(1 - \frac{\sigma \omega_B^2}{2 \omega_G^2}\right) \left(1 - \frac{h \sigma}{2}\right) + \frac{\sigma \omega_B^2}{2 \omega_G^2} \left(1 - \frac{\sigma}{h}\right),$$  \hspace{1cm} (46)$$
is greater than $p^{\text{base}}$ for all $h < \sigma$. Therefore, $h < 1$ is a better predictor of identifying player $G$, and predictability is maximized when $h = \sigma$.

Case 3: $h \sigma \geq 1$ and $\sigma/h < 1$, and $\omega_G^3 = \sigma/h$ and $\omega_B^3 = 1/(h \sigma)$, so condition $\sigma \omega_B^3 > \omega_G^3$; Polborn’s second case, occurs.

Case 3, Rule 1: The probability that identify player $G$ is now

$$p^{3.1} = \left(\frac{\omega_G^3}{2 \sigma \omega_B^2}\right),$$  \hspace{1cm} (47)$$
and is less than $p^{\text{base}}$ for all $h > 1/\sigma$.

Case 3, Rule 2: The probability that identify player $G$ is now

$$p^{3.2} = \frac{\omega_G^3}{2 \sigma \omega_B^2} \frac{1}{2h \sigma} + \left(1 - \frac{\omega_G^3}{2 \sigma \omega_B^2}\right) \left(1 - \frac{\sigma/h}{2}\right),$$  \hspace{1cm} (48)$$
and is greater than $p^{\text{base}}$ for all $h > 1/\sigma$. However, the probability is not greater than $p^{1.2}$ or $p^{2.2}$ when $h = \sigma$. \(\square\)

**Proof of Proposition 4:**

Since the contest designer only considers maximizing the second period effort without commitment at time period 2, and not time period 1 as in the commitment case, then only the first case ($c = 1$) is relevant.

$$E_2 : \max_h \left(1 - \frac{\sigma \omega_B}{2 \omega_G}\right) (\sigma hv) + \frac{\sigma \omega_B}{2 \omega_G} \left(\frac{\sigma}{h} v\right), \text{ subject to, } \sigma < h < 1/\sigma,$$  \hspace{1cm} (49)$$
where

$$\omega_G = v + ((1 - h \sigma) - (1 - \sigma/h)) v,$$

and

$$\omega_B = v + (0 - 0) v.$$

The optimal handicap without commitment ($\hat{h}^*$) satisfies the first order-condition

$$\frac{1}{2} \sigma (h^* + 1) = (h + \sigma - \sigma h^2)^2,$$
and is less than the optimal handicap with commitment $h^* = \left(1 - \sigma + (1 - 2\sigma + 5\sigma^2)^{1/2}\right)/2\sigma$ for all $\sigma$. □

6 References


