Favoring the Winner of Loser: Handicapping in Repeated Contests

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Abstract

Should a firm favor a weaker or stronger employee in a contest? Despite a widespread emphasis on rewarding the best employees, managers continue to tolerate and even favor poor performers. Contest theory reveals that evenly matched contests are the most intense, which implies that a contest designer can maximize each player’s effort by artificially boosting the underdog’s chances. We apply this type of “handicapping” to a two-period repeated contest between employees, in which the only information available about their abilities is their performance in the first-period. In this setting, employees are strategic and forward looking, such that they fully anticipate the potential impact of the first-period contest result on the second period contest, and thus adjust their behaviors accordingly. The manager also incorporates these strategic behaviors of employees when determining an optimal handicapping policy. If employees’ abilities are sufficiently different, favoring the first-period loser in the second period increases total effort over both periods. However, if abilities are sufficiently similar, we find the opposite result occurs: total effort increases most in response to a “reverse handicapping” strategy that favors the first-period winner. We also show that reverse handicapping maximizes effort even with perfect information.

Keywords: game theory, contests, handicap, reverse handicap, racheting, asymmetric information
1 Introduction

Firms often face a tough decision about whether to invest in laggards or reward their top performers. This fundamental question in management still does not have gained a consensus answer. Many experts recommend that managers should reward their top employees and avoid coddling weaker employees with lower standards; for example, best sales agents often receive more training and obtain more back-office resources (Farrell and Hakstian 2001). Indeed, $1.3 billion in training expenses are “devoted to grooming leaders” and managers are often told to look for top performers to receive specialized learning opportunities (Kranz 2007). However, more than 60% of employees surveyed indicated that their managers tolerate poor performers implying that top performers were not being recognized (Sales and Marketing Management 2007). Furthermore, nearly two-thirds of managers claim they spend a majority of their time dealing with and helping poor performers (Sales and Marketing Management 2003, 2004a). Such results may be strategic in the sense that handicapping (i.e., favoring) a weak performer might improve his chances of success, encouraging him to work harder.¹ Not only would this improve performance by raising the skill level of employees, but would also increase the general competition among employees by improving the performance of weak employees (Farrell and Hakstien 2001; Sales and Marketing Management 2004a). It is therefore unclear what the best strategy is for the manager.

To address the issue of whether a firm should favor weaker or stronger employees, we adopt a contest theory approach. Managers often reward employees based on overall evaluations over a certain period of time, rather than on a narrowly defined sales contest, (which is a short-term temporary monetary incentive program for salespeople). In this study, we refer to this entire evaluation time period as the contest. As such, we broadly define a contest as any competition between employees, including competition for limited support resources and promotion to higher ranks. We refer to several reward systems firms use to motivate and encourage employees to expend their efforts such as monetary bonuses, promotions or more subtle forms of various privileges (e.g., less administrative responsibilities, more back-office support or more training opportunities) as the contest prize.

Contests typically involve two or more employees competing for a single prize, and the

¹We use the paradoxical terminology of handicapping, as popularized by its usage in association with golf, which implies that a higher handicap helps a player.
employee who performs best usually wins (i.e., winner-take-all contest). By adding this winner-take-all component, firms can induce a significant increase in effort. However, the effect is unclear when a manager faces heterogeneous employees who differ in their abilities. Weaker employees often recognize their small chance of winning and thus have little motivation to increase their effort (Hart et al. 1989, Murphy et al. 2004, Corsun et al. 2006). Consequently, stronger employees, anticipating less competition, also may respond with limited changes to their behavior or even lower effort. In this case, the contest fails to properly motivate the employees or meet the goals of the manager.

Previous research in economics and marketing suggests guidelines for contests. For example, contest theory suggests leveling the playing field by granting an advantage to a weaker employee, which may increase overall effort (Lazear and Rosen 1981, Baik 1994, Baye et al. 1993, Liu et al. 2007). The advantage increases the weaker employee’s chances of success, similar to giving a weaker golfer a “handicap”, which should make the contest more intense. Therefore, helping the weaker employee can make all employees compete harder to win the contest.

However, in practice, some contests actually favor the stronger employees. For example, successful sales agents often receive more lucrative territories or product lines (Skiera and Albers 1998), are assigned less administrative responsibilities, obtain more back-office resources, or have more training (Krishnamoorthy et al. 2005; Farrell and Hakstian 2001). Similar examples exist in other fields. Successful researchers tend to have more grant opportunities (Che and Gale 2003), winners of the regular season receive home field advantage in a sport’s postseason, and winners of early speed trials gain the most favorable position in car races (Mastromarco and Runkel 2006). Similarly, by winning previous contests, well-established firms and incumbent politicians often enjoy easier runs in subsequent contests. In most cases, such favoring the winner over the loser (i.e., "reverse handicapping") seems to contradict the principle of maximizing effort by evening the playing field.2

These existing models also assume that the contest designer can perfectly distinguish between high and low ability employees. However, most contest designers in reality face uncertainty about employees’ abilities and therefore must base their assessment, in part, on employees’ past performances in previous periods. Past performance clearly offers a strong, albeit noisy, signal

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2This terminology is introduced by Che and Burguet (2004) who find that favoring the better player in procurement auctions can be optimal in an environment with corruption. In sailing terminology "reverse handicapping" refers to a standard handicapping strategy in which boats in a race start at different times rather than together.
of ability.

In this article, we consider a two-period repeated contests (Amaldoss and Rapoport 2009) under uncertainty where the contest designer (i.e., manager) faces heterogeneous employees but does not know the exact abilities of each individual employee. The manager only receives a noisy signal of employees’ abilities through first-period contest results. In the second-period contest, the manager can assign a handicap favoring the winner or loser of the previous contest. However, this handicapping policy might create a new incentive problem in a dynamic setting where the two contests are linked through the handicapping policy. For example, if employees anticipate that winning a current contest will hurt them in the future, they have less incentive to win the first contest. This “ratcheting,” by which employees modify their efforts in the current period to alter incentives in future periods, is a common problem for workforce management (Chung et al. 2010, Misra and Nair 2011, Weitzman 1980).

Favoring the loser of the first contest thus creates an incentive problem in which both players reduce their attempts to win in the first period so they can take advantage of second-period handicapping. On the other hand, favoring the winner of the first contest would also create another type of incentive problem, because the winner no longer has to work as hard in the second contest. The manager should weigh the trade-offs of these two different policies (favoring the winner vs. favoring the loser) to maximize the employees’ total effort.

Here, we adopt a Tullock (1980) contest model, which uses the standard ratio-form contest success function. The model clearly demonstrates that employees’ incentives change over time with different handicapping policies (i.e., favoring weaker vs. stronger employees) and fully considers the strategic effects of the employees (i.e., ratcheting). In particular, if abilities are sufficiently similar, surprisingly favoring the first-period winner in the second period increases total effort over both periods – rewarding the top performer is optimal. However, if abilities are sufficiently different, the opposite result holds, and total effort is maximized by adopting a handicap policy that favors the first-period loser – investing in laggards is optimal. As such, the model suggests a clear handicapping policy guideline for the manager when faced with a heterogenous workforce with uncertain abilities in a contest environment.

More specifically, we first find that when the relative ability gap between weaker and stronger employees is sufficiently large, the standard handicapping policy of favoring the loser of the first period will increase total effort. However, when the ability gap is small, the opposite result holds, and total effort is maximized by favoring the first-period winner. As such, the model suggests a clear handicapping policy guideline for the manager when faced with a heterogenous workforce with uncertain abilities in a contest environment.
The optimal period is optimal. Even though the second-period handicapping decreases the total first period effort, the employees’ strategic consideration for the second-period handicap overcomes this effort loss in the first period. This is because the second period handicapping hurts the stronger employee’s first-period incentives more than it hurts the weak employees’ first-period incentives. In this way, the first-period contest becomes more equitable through the second-period handicap. This mitigates the negative impact on total first-period effort for very large ability differences.

On the other hand, when employees’ abilities are sufficiently similar, we find that a reverse handicapping of favoring the winner of the first period contest is optimal. With little difference in their abilities, the extent of advantage the winner of the first contest receives (the reverse handicap) is also small. Accordingly, the loser of the first period contest still has a chance to win in the second period even with a small disadvantage, and thus experiences sufficient incentive to exert effort. Hence, the increase in effort in the first period more than offsets the effort loss in the second period when employees’ abilities are sufficiently similar.

The rest of this article is organized as follows: Sections 2 describes the related literature. Section 3 and 4 present the static and dynamic models of contests, respectively. We also provide extensions to our model in Section 5, and we conclude in Section 6.

2 Literature Review

The theoretical foundations of contests stem from the economics literature. Since Tullock (1980) published his seminal contest model in which players vie for a single prize through the expenditure of their resources (sometimes called a Tullock or ratio-form contest), several models have been proposed for different types of contests (Lazear and Rosen 1981, Moldovanu and Sela 2001). In particular, we follow Meyer (1991, 1992) to determine how the uncertainty about employee types affects a manager’s handicapping policy. Meyer (1991, 1992) analyzes a multi-period contest under uncertainty with symmetric (i.e., equal ability) players using a Lazear-Rosen (1981) difference-form contest, in which success is a function of the difference in (ability-adjusted) effort levels. The Lazear-Rosen (1981) model differs from most models in the contest literature, in that (1) it does not follow the standard Tullock contest form (which uses the ratio-form success function) and (2) it does not use a linear cost function for effort. In our model, we instead adopt the Tullock contest model and attain pure strategy equilibria where both sides exert effort. This is not possible in difference-form asymmetric contests with nonlinear costs (Hirshleifer 1989,
In this sense, our study extends the robustness of Meyer’s (1991, 1992) results of symmetric case to a Tullock (1980) ratio-form contest and uncovers new results in the asymmetric case. Meyer (1992) shows that in a promotion setting, giving an advantage to the winner increases overall effort (or equivalently minimizes the cost of prizes to induce the same effort). This result is, however, limited to only the symmetric players case, and we extend it to investigate the incentive problem in a two-period dynamic contest between two asymmetric employees (i.e., individuals with different abilities) who strategically choose their efforts in response to the handicapping policy.

The model of contest has been extensively applied in various contexts outside of sales contests. For example, Tsoulohas et al. (2007) use a contest model to study the issue of employee promotion selection and show when it is optimal to handicap insiders or outsiders for CEO selection. Horsky et al. (2010) investigate the advertising agency selection problem using a contest model and Harbaugh and Ridlon (2010) apply the contest model to the all-pay auction setting. In particular, Harbaugh and Ridlon (2010) have a similar theme and structure with the current model in that both investigate the issue of optimal handicapping policy between asymmetric players in a two-period contest setting. However, unlike Harbaugh and Ridlon (2010) who only finds support for handicapping the loser in the first period for all ability differences (i.e., the handicapping policy always maximizes total bids in the all-pay auction setting), our model clearly identifies the conditions in which reverse-handicapping (favoring the winner) could be optimal. Furthermore, the bid equilibrium in Harbaugh and Ridlon (2010) can only be found in mixed strategies which are not realistic in most contest settings. Our model overcomes this inconsistency of a mixed strategy equilibrium and provides richer results as they relate to competition between asymmetric employees.4

Several contests have also been examined with company objectives other than effort maximization, such as to boost employee morale (Murphy et al. 2004), increase sales (Brown and Peterson 1994), improve customer satisfaction metrics (Hauser et al. 1994), increase accuracy in

4Harbaugh and Ridlon (2010) examines the auction setting where the expected payoff to the weaker player is zero and, therefore, there is no strategic effect of the handicapping policy on the weaker player in the first period (since his payoff cannot be lower than zero). Hence, their model does not fully incorporate the tension between two different incentives created by different handicapping policies, which is the focus of the current paper. Thus, their model cannot find the pure strategy equilibrium or the conditions under which reverse-handicapping can be optimal.

In marketing, most theoretical research focuses on the issue of optimal contest design. Kalra and Shi (2001) is the first paper to have examined sales contests from a game theoretical perspective. They identify specific conditions in which the optimal contest design structure should include multiple prizes at varying levels to induce greater effort by all salespeople. Krishna and Morgan (1998) further show that winner-take-all contests are optimal when contestants are risk-neutral. Lim (2010) uses a behavioral economics model to demonstrate that a contest with a higher proportion of winners than losers can yield greater effort than one with fewer winners under certain conditions. Finally, Lim et al. (2009) show empirically using laboratory and field experiments that the prize structure of a sales contest indeed affects the effort of contestants: Effort levels increase when the number of prize winners is greater than one.

In contrast, we do not address the issue of optimal structure of contests or their prizes. Instead, in the context of the common winner-take-all structures (Krishna and Morgan, 1998), we try to answer management’s fundamental dilemma whether to favor the stronger or weaker employees using a two-period repeated contests model.

3 One-Period Static Model

3.1 Preliminary: Standard Contest with One Period

Consider a simple one-period contest between two employees. In our model setting, a contest can be thought of as a broader competition between agents, including competition for limited support, resources or promotion to higher levels of position, rather than a particular sales contest. These employees are heterogeneous in their abilities, denoted by the parameters, $a_b > 0$ and $a_g > 0$, where $a_b \leq a_g$, without loss of generality. Therefore, the good player, employee $G$, is stronger and more effective than the bad (or weaker) player, employee $B$, in ability. They compete to win a single prize by exerting effort that increases their probability of winning. Let $e_b$ and $e_g$ represent the efforts exerted by employee $B$ and employee $G$, respectively. We assume that efforts are unobservable by the manager, who only identifies a winner of the contest. The ability parameters directly influence the effectiveness of converting effort into performance. Thus, lower ability leads to lower performance, all else being equal. The probability of $B$ winning
the contest is his effort relative to the total effort of both B and G. We specify employee B’s contest success probability \(s_B\) by following the standard Tullock (1980) ratio form contest success function

\[
s_B = \frac{a_b e_b}{a_b e_b + a_g e_g}, \tag{1}
\]

where \(\partial s_B / \partial e_b > 0\) and \(\partial^2 s_B / \partial e_b^2 < 0\) for all \(e_b\), such that extra effort by player B increases his probability of winning at a decreasing rate. Furthermore, the probability of winning decreases with his rival’s effort while the ability parameter increases the probability of success as it becomes greater. In other words, as employee B becomes stronger, the more likely he is to win the prize.

For simplicity, we define the relative ability, \(\sigma = \frac{a_b}{a_g}\), and \(0 < \sigma < 1\) since \(a_b \leq a_g\). The relative ability \(\sigma\) is common knowledge, and employees know their own abilities. However, the manager only receives information on relative ability and does not know the exact type of each employee. Employees compete for a privately valued prize, denoted \(v_B\) and \(v_G\), and have unit costs in effort.

Employee B’s utility function is written as

\[
U_B = s_B \cdot v_B - e_b = \left( \frac{a_b e_b}{a_b e_b + a_g e_g} \right) v_B - e_b
\]

\[
= \left( \frac{\sigma e_b}{\sigma e_b + e_g} \right) v_B - e_b.
\]

The first-order condition for maximizing his utility is \(\frac{\partial U_B}{\partial v_B} = 1\). Similarly, employee G’s utility function is \(U_G = (1 - s_B) \cdot v_G - e_b\), and the first-order condition for maximizing her utility is \(\frac{\partial U_G}{\partial v_G} = 1\). It is standard to find that the equilibrium efforts for employee B and G are, respectively,

\[
e_b^* = \frac{\sigma v_G}{(v_G + \sigma)^2}, \quad e_g^* = \frac{\sigma v_B}{(v_B + 1)^2}, \tag{3}
\]

where \(\frac{\partial e_b}{\partial v_B} > 0\) and \(\frac{\partial e_g}{\partial v_G} > 0\) for all \(v_B, v_G > 0\), such that equilibrium efforts increase in their own valuation of the prize.

The standard Tullock contest results are conveyed in the following lemmas (Baik 1994, Nti 1999), which characterize the behavior of the employees in a contest.

**Lemma 1** The ratio of efforts, \(\frac{e_b}{e_g}\), is equal to the ratio of valuations, \(\frac{v_B}{v_G}\).

**Lemma 2** Difference in abilities reduces total efforts by both employees \((e_b + e_g)\).

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5The second-order sufficiency condition for B (and G), \(\frac{\partial^2 U_B}{\partial e_b^2} = -\frac{2e_g \sigma^2}{(e_g + e_b)^3} v_B < 0\), holds.
Lemma 1 implies that when valuations are identical \( v_G = v_B \), the equilibrium levels of efforts are identical, regardless of ability differences (i.e., \( e_b = e_g \)). That is, the relative efforts are independent of any ability differences. Any change in relative valuation, however, has an equivalent change in relative effort. At the same time, Lemma 2 asserts that total efforts depend on ability differences, and even are decreasing as ability differences become greater. Lemmas 1 and 2 are easily verified with Equation (3). As the employees’ abilities become increasingly different, total effort decreases, which reduces the effectiveness of the contest, though the relative efforts by the two employees remain constant. These lemmas have important implications for our main analysis.

Inserting the equilibrium efforts (Equation 3) into the utility functions, we find for employee B,

\[
U_B = \frac{\sigma v_B}{(\sigma v_B + v_G)} v_B - \frac{\sigma v_G v_B^2}{(\sigma v_B + v_G)^2} \]

and for employee G,

\[
U_G = \frac{v_G}{(\sigma v_B + v_G)} v_G - \frac{\sigma v_G^2 v_B}{(\sigma v_B + v_G)^2} \]

In this basic contest, both employees exert positive efforts and earn positive utility, even with the differences in their abilities.\(^6\) The manager still benefits from the induced extra efforts. Next, we examine the impact of handicapping on these results.

### 3.2 Contest Handicapping

Let \( h \) be the handicap policy associated with reducing the relative ability gap, \( \sigma = \frac{a_b}{a_g} \leq 1 \), between the players. The handicap, \( h \geq 0 \), has a multiplicative effect on the ability parameter, \( \sigma \), (i.e., \( h\sigma \)), such that when \( h > 1 \), handicapping reduces asymmetry, and when \( h < 1 \), reverse handicapping amplifies the asymmetry. For simplicity, we assume that heterogeneity only ap-

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\(^6\)Note that at the limit, as \( \sigma \) approaches zero, efforts for both employees decrease to zero. Hence, in the case of \( \sigma = 0 \), the stronger employee wins the contest automatically, which is equivalent to presenting an award for achievement without having a contest at all.
pears in ability, and valuations are set to $v_G = v_B = v$.\footnote{Employee heterogeneity can be expressed in terms of cost of effort, prize valuation, and ability. We assume these costs and valuations are constant between players and allows only for ability to differ. See Baye and Hoppe (2003) for a discussion of the strategic equivalence of contests with asymmetric costs, valuations, and abilities.} Recall from Lemma 1 that efforts are the same when valuations are identical regardless of ability asymmetry.

From Equations (3), we express total effort as

$$ e^{Total} = e_b + e_g = 2\frac{\sigma h}{(\sigma h + 1)^2} v. \tag{6} $$

The objective of the manager in our setting is to maximize company profit which is a function of total employee effort $\pi_M = \psi (e^{Total}) - v$, where $\psi' > 0$, $\psi'' < 0$. Since, total effort increases overall firm profit ($\frac{\partial \psi}{\partial e^{Total}} > 0$), the manager only needs to find the optimal $h$ that maximizes the total employee efforts.

Because any asymmetry between players reduces total contest effort, the manager has an incentive to make the contest more equitable by giving an advantage to the weaker player. This handicap not only increases the perception that the contest prize is more attainable for the weaker player, but it also intensifies the competition between the employees. When the manager knows the abilities of the employees, she simply handicaps the weaker employee by directly altering the ability parameter (Tullock 1980).

**Lemma 3** *When the manager knows the employees’ types, the optimal handicapping policy is $h^* = \frac{1}{\sigma} > 1$.*

This is a well-known result derived from Tullock (1980). When the manager has perfect information about the employees’ types (i.e., the identity of the weaker employee is known), in a one-shot static game with a given ability difference $\sigma$, the optimal policy is to handicap the weaker employee by the exact reciprocal amount of the ability difference, $h^* = \frac{1}{\sigma}$, equalizing the two employees’ abilities. Not surprisingly, this handicap maximizes total effort, such that it is equal to total effort when employees are symmetric.

### 3.3 Handicapping under Uncertainty

Let us now consider a more realistic situation in which the manager may know $\sigma$ based on an expectation from the distribution of abilities (in other words, the manager knows the distribution of abilities, but does not know the exact location of each individual on the distribution). For example, teachers may have expectations of performance from poor performers and top performers.
in class from their own experience (for example, top performers may historically score in the 90th percentile while poor performers would score in the 10th percentile in SAT), but students’ types are revealed only after performance evaluations. Similarly, in hiring new employees, the firm has only an expectation of each level of weak and stronger employee abilities (an thus, their ability difference $\sigma$), but cannot determine the identity of the weaker employee or the stronger employee. In this case, closing the ability gap by applying a handicap to one of the players might reduce total effort. Without information about the identity of the weaker employee, the handicap allocation would be random and might erroneously apply the handicap to the stronger player, further increasing the ability disparity.

In practice though, managers receive multiple noisy signals about employees’ types, such as their absenteeism, project completion, prior performance evaluations and recommendations, or previous contest results. We model the uncertainty of such a noisy signal as $p$, such that the signal is incorrect with probability $p$ and correct with probability $(1 - p)$. In other words, with probability $p$, employee $B \ (G)$ is incorrectly identified as stronger (weaker), and with probability $(1 - p)$, employee $B \ (G)$ is correctly identified as the weaker (stronger) employee.

The manager implements the handicapping policy prior to the contest and grants a handicap to an employee, based on the ability parameter $\sigma$ and the signal parameter $p$. The handicap $h \geq 0$ again has a multiplicative effect on the ability parameter, but the actual implementation differs because uncertainty about each employee’s type remains. The impact of handicapping policy $h$ on the ability gap is either $\sigma h$, if the handicap is correctly applied to the bad type employee, or $\frac{\sigma}{h}$, if it is incorrectly applied to the good type employee. For example, when $\sigma = \frac{1}{2}$, employee $B$ must exert twice as much effort as employee $G$ to equal his chance of winning the contest without any handicap policy. If a handicap policy of $h = \frac{1}{\sigma} = 2$ is correctly applied to the weaker employee $B$ (i.e., $\sigma h = 1$), $B$ only has to equal $G$’s effort to achieve an equal chance of winning. However, $B$ would have to exert four times as much effort as employee $G$ if the handicap were incorrectly applied to $G$ (i.e., $\frac{\sigma}{h} = \frac{1}{4}$).

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8 Only after some time periods of relationship, the manager can determine the identity of the stronger or weaker employee. However, given the lack of track record at the time when the firm hires new employees, it is hard to figure out which one would turn out to be a stronger employee (it might depend on several factors such as fit, ability and luck). It is also common for employees to have a greater knowledge about their coworkers than the manager through their personal interactions (Sales and Marketing Management, 2004b). We relax this assumption that the manager has imperfect information about their employees in the extensions section and see the case when the manager has perfect information about employees’ types.
Given a handicapping policy $h$ and uncertainty $p$, we can calculate the equilibrium efforts under handicapping from Equation (3). With probability $1 - p$, employee $B$ is (correctly) identified as the weaker employee and receives a handicap of $h$. Equilibrium levels of effort for both employees then are

$$e^t = e^t_b = e^t_g = \frac{\sigma h}{(1 + \sigma h)^2} v.$$  

(7)

Since $v_G = v_B = v$, both employees exert the same amount of effort $e^t$ in equilibrium (Lemma 2), where the superscript ‘$t$’ represents their ‘true’ identity.

But with probability $p$, employee $B$ is identified (incorrectly) as the strong employee, and the handicap is erroneously applied to the truly stronger employee $G$. Hence, employee $B$ implicitly receives a reverse handicap of $\frac{1}{h}$, and the equilibrium levels of effort for both employees are

$$e^f = e^f_b = e^f_g = \frac{\sigma h}{(\sigma + h)^2} v.$$  

(8)

Again, the equilibrium level of effort, $e^f$, is the same, irrespective of employee type, where the superscript ‘$f$’ represents their ‘false’ identity.

Total expected equilibrium effort given a signal $p$ and handicapping policy $h$ for all $\sigma$ is

$$E(e^{Total}) = p \left(2e^f\right) + (1 - p) \left(2e^t\right).$$  

(9)

Thus, we can characterize the effects of $p$ on effort and choice of $h$ in the following proposition, which is our first result.

**Proposition 1** When $p$ is sufficiently small, a handicapping policy ($h > 1$) is optimal for all ability differences ($\sigma < 1$). Moreover, this optimal handicapping policy is strictly smaller than $\frac{1}{\sigma}$ ($h < \frac{1}{\sigma}$).

**Proof.** See the Appendix. 

When the manager receives a sufficiently informative signal (i.e., small $p$), the optimal policy for her is to give an advantage to the perceived weaker employee ($h^* > 1$), which is consistent with existing literature in the perfect information case (Lemma 3). When $p = 0$, Equation (9) becomes identical to Equation (6) under perfect information case, and the contest designer can identify the abilities of the players precisely.

The manager thus benefits from the signal, even a noisy one, and she can act upon it by applying a handicap to the employee who appears weak. However, this handicapping policy does not fully compensate the ability difference due to the uncertainty. The optimal handicapping
policy under uncertainty is strictly smaller than that of perfect information case, where the handicapping policy precisely equalize the two employees’ abilities by fully compensating the exact amount of ability difference ($\frac{1}{2}$).

In the next section, we extend our basic static model to the case of a two-period repeated game, and explicitly model the source of signal $p$ as the results of a first-period contest; that is, as the endogenous outcome of a game between strategic, forward-looking employees.

4 Dynamic Model\(^9\)

4.1 Standard Contest with Two Periods

Second Period

Suppose now that the manager observes a two-period repeated contests in which the values of the prize for each employee are identical and do not vary between periods, such that $v_G = v_B = v$ and $v_1 = v_2 = v$.\(^{10}\) Prior to the first-period contest, the manager commits to a handicapping policy $h$, which is observable to the employees. In practice, firms explicitly proclaim the way a winner is selected with well-defined criteria as well as a publicized benefit for the winner or the loser in the future (we examine the case of no commitment in the model extension). In the first period, the employees compete in a contest, and the manager gains information about the employees’ types by observing who wins or loses. This signal is still noisy because the success function, or winning probability, of the first-period contest is stochastic (see Equation (1)) and more importantly, employees may strategically adjust their behaviors. Using the information about who wins the contest in the first period, the manager assesses, with some probability, who is the weak employee and assigns the pre-determined $h$ for the second-period contest. Hence, the manager treats the result of the first period contest as the signal about employees’ types.

Here, employees are strategic and forward looking, such that they fully anticipate the potential impact of the first-period contest result on the second period contest, and thus adjust their behaviors accordingly. The manager also considers the employees’ strategic behaviors when

\(^9\)It is important to note that the dynamic model in the research studies a two-period repeated contests, not a contest that lasts over two periods. In other words, we study dynamic relation of two contests, but not a dynamic contest.

\(^{10}\)By keeping the values of the each contest the same, we can focus on the direct effect of $h$ on the equilibrium outcome. However, an optimal $h$ is clearly affected by differences in valuations between periods.
determining an optimal handicapping policy.

We accommodate this dynamic incentive on employees’ behaviors in both periods, which distinguishes this section from the previous section. Note that the static result of imperfect information (Proposition 1) in the previous section serves as a benchmark for the second period result of the current dynamic game. In the static game, the uncertainty of the signal is exogenously given as \( p \) whereas we endogenize the source of signal \( p \) as the outcome of strategic behaviors of employees in this section.

We start by considering the second period. Instead of receiving an exogenous signal with uncertainty \( p \), the manager receives the endogenous signal from the first-period contest as a win or loss, with probability equal to the first period success function. Since \( v_G = v_B = v \), the second-period equilibrium efforts of both employees are identical (Lemma 1). However, given the handicapping policy \( h \), the exact level of effort only depends on which employee won the first-period contest. If the weaker employee \( B \) wins the contest in the first period, he is believed (incorrectly) to be the stronger employee, and the handicap is erroneously applied to the truly stronger employee. In this case, the equilibrium effort in the second period will be \( e_f^2 = e_b^f = e_g^f = \frac{\sigma h}{(\sigma + h)^2} v \), from Equation (8). But, if employee \( B \) loses the contest in the first period, he will be correctly identified as the weaker employee and receive a handicap of \( h \). Equilibrium levels of effort for both employees in the second period are hence, \( e_f^2 = e_b^f = e_g^f = \frac{\sigma h}{1 + \sigma h} v \), from Equation (7).

To calculate the second-period success function, we insert these equilibrium efforts into Equation (1). Thus, the second-period success functions for employee \( B \) are conditional on the outcome of the first-period contest. If \( B \) loses \((L)\) in the first period (and thus, equilibrium effort is \( e_f^L = \frac{\sigma h}{1 + \sigma h} v \)), the contest success function for employee \( B \) is

\[
s_B^L = \Pr(B \text{ wins} \mid B \text{ loses first period contest}) = \frac{\sigma h}{1 + \sigma h}. \tag{10}
\]

If he wins \((W)\) in the first period (and thus, equilibrium effort is \( e_f^W = \frac{\sigma h}{(\sigma + h)^2} v \)), the contest success function for employee \( B \) is

\[
s_B^W = \Pr(B \text{ wins} \mid B \text{ wins first period contest}) = \frac{\sigma}{\sigma + h}. \tag{11}
\]

Employees \( B \)’s and \( G \)’s second-period expected utilities are, respectively,

\[
E(U_{B,2}) = s_B \cdot \left[ (s_B^W) v - e_f^W \right] + (1 - s_B) \cdot \left[ (s_B^L) v - e_f^L \right], \tag{12}
\]

\[
E(U_{G,2}) = (1 - s_B) \cdot \left[ (1 - s_B^W) v - e_f^W \right] + s_B \cdot \left[ (1 - s_B^L) v - e_f^L \right],
\]
where $s_B$ is the probability that employee $B$ wins in the first period contest ($s_B = \frac{\sigma e_b}{\sigma e_b + e_g}$ from Equation (1)).

**First Period**

Now, we turn to the first-period utility to understand the employees’ optimal effort decisions. Since employees are strategic and forward-looking, they incorporate the fact that their first-period efforts affect their second-period utility. Thus, employee $B$ chooses an effort level in the first period that maximizes his total expected utility:

$$E(U_B) = s_B v - e_b + \delta \left\{ s_B \cdot \left[ (s_{B|W}) v - e^f_2 \right] + (1 - s_B) \cdot \left[ (s_{B|L}) v - e^f_2 \right] \right\}$$  \hspace{1cm} (13)$$

where $\delta$ is the discount factor, which we normalize to 1 for simplicity. The first-order condition is

$$\frac{\partial s_B}{\partial e^f_b} v + \frac{\partial s_B}{\partial e^f_b} \left( (s_{B|W}) v - e^f_2 \right) - \frac{\partial s_B}{\partial e^f_b} \left( (s_{B|L}) v - e^f_2 \right) = 1$$  \hspace{1cm} (14)$$

where

$$\omega_B = v + \left\{ (s_{B|W}) v - e^f_2 \right\} - \left\{ (s_{B|L}) v - e^f_2 \right\}$$

$$= v + v \left[ \frac{\sigma}{\sigma + h} \right]^2 - \left[ \frac{\sigma h}{1 + \sigma h} \right]^2.$$  

Here, $\omega_B$ captures employee $B$’s implicit value of winning the first-period contest. The first term ($v$) represents the utility from the first-period contest, and the second $\left[ (s_{B|W}) v - e^f_2 \right]$ and third $\left[ (s_{B|L}) v - e^f_2 \right]$ terms represent the extra future value (or cost) of winning the first-period contest, depending on $\sigma$ and $h$.

Similarly, the first-order condition for maximizing employee $G$’s total utility function is

$$\frac{\partial (1 - s_B)}{\partial e^f_b} \omega_G = 1 \Leftrightarrow \left( \frac{\sigma e_g}{(\sigma e_b + e_g)^2} \right) \omega_G = 1,$$  \hspace{1cm} (15)$$

where

$$\omega_G = v + \left\{ (1 - s_{B|L}) v - e^f_2 \right\} - \left\{ (1 - s_{B|W}) v - e^f_2 \right\}$$

$$= v + v \left[ \frac{1}{1 + \sigma h} \right]^2 - \left[ \frac{h}{(\sigma + h)} \right]^2.$$  

\[11\text{It is easy to check that the second-order condition, } \frac{\partial^2 U_B}{\partial e^f_b} < 0, \text{ is still satisfied.} \]
Here, $\omega_G$ represents employee $G$’s implicit value of winning the first-period contest.

From Equations (14) and (15), we find that first period equilibrium efforts are

$$e^*_b = \frac{\sigma \omega_G}{(\frac{\omega_G}{\omega_B} + \sigma)^2}, \text{ and } e^*_g = \frac{\sigma \omega_B}{(\frac{\omega_B}{\omega_G} + 1)^2}. \quad (16)$$

As a result, the probability of employee $B$ winning in the first period is simply

$$s_B = \frac{\sigma \omega_B}{\sigma \omega_B + \omega_G}. \quad (17)$$

Note that the implicit valuations in the first period, $\omega_B$ and $\omega_G$, are independent of first period efforts ($e_b, e_g$). That is, they are treated as exogenous prizes just as in the static, single-period contest case. However, a change in handicapping policy $h$ affects the effort levels of both employees in the first period by either increasing or decreasing the implicit value of winning in the first period ($\omega_B, \omega_G$). This is the strategic (indirect) effect of $h$ on the first period efforts.

The following proposition shows how differently a handicapping policy affects the implicit value of winning in the first period for different types, which ultimately affects their effort levels.

**Proposition 2** The value of winning the first period contest decreases in handicapping policy $h$ ($\frac{\partial \omega_G}{\partial h} < 0, \frac{\partial \omega_B}{\partial h} < 0$). Moreover,

1. Under a handicapping policy (i.e. when $h > 1$), the value of winning the first-period contest is such that $\omega_G < \omega_B < v$.

2. Under a reverse handicapping policy (i.e., when $h < 1$), the value of winning the first-period contest is such that $\omega_G > \omega_B > v$.

3. Without a handicapping policy (i.e., $h = 1$), the value of winning the first-period contest converges to the static single-period case for both employees: $\omega_G = \omega_B = v$.

**Proof.** See the Appendix.

The proposition basically suggests that when $h$ becomes larger (i.e., the manager favors the loser of the first period more), the value of winning the first period decreases for both types ($\frac{\partial \omega_G}{\partial h} < 0, \frac{\partial \omega_B}{\partial h} < 0$) since the loser can benefit in the second period from the handicapping policy $h$. This is the strategic effect of $h$ due to the dynamic relationship between two-period contests. What is more interesting and surprising is that the value for the good employee is
greater or smaller than that of the bad employee depending on the handicapping policy (i.e. whether $h < 1$ or $h > 1$).

Under a handicapping policy (i.e., $h > 1$), the loser of the first period contest still benefits in the second period, and consequently losing the first-period contest is less devastating as it is in the static game for both employees: $\omega_G < v$ and $\omega_B < v$. Because of this, both employees may modify their efforts by holding back in the first period. This dynamic arises from the ratchet effect identified in previous literature (Freixas et al. 1985, Weitzman 1980). Moreover, this decrease in value is greater for the stronger employee who is more likely to win the contest and $\omega_G < \omega_B < v$ for $h > 1$. This is because the good employee $G$ benefits more from losing the first period contest since favoring the employee $G$ in the second period makes the contest even more asymmetric rather than less. On the other hand, employee $B$ is still more likely to lose in the second period contest even with the advantage from handicapping ($h$) since the handicapping does not fully compensate the ability difference due to the uncertainty (see Proposition 1). Hence, employee $B$ would value winning the first period contest more than employee $G$.

On the other hand, under a reverse handicapping policy ($h < 1$), the value of winning the first-period contest is higher than the static single-period case for both employees: $\omega_G > \omega_B > v$ for $h < 1$. By rewarding the winner of the first period with an advantage in the second period, the value of the first period contest is larger and, therefore, winning the first period contest is more valuable than in the static case. More importantly, the value of winning for the good employee is greater than that of the bad employee when $h < 1$. Again, this indirect effect of reverse handicapping is more pronounced for the stronger employee $G$ than the weaker employee $B$ since employee $G$ benefits more from the increased asymmetry in the second period contest while the weaker employee $B$ is still more likely to lose even with reverse handicapping policy in the second period.

Figure 1 below illustrates the relationship between the handicapping policy ($h$) and the value of winning the first period contest (when $v = 1$ and $\sigma = 0.5$).

The implicit value of winning varies for each type under different handicapping policy and thus affects their effort levels differently. From Equation (16), it is clear that the direct effect of its own implicit value of winning is to increase the first period effort: $\frac{\partial e_b}{\partial \omega_B} > 0$ and $\frac{\partial e_g}{\partial \omega_G} > 0$. The higher the stake, the more they exert their effort.

However, changes in rivals’ efforts are more ambiguous. The indirect effect of implicit value
of winning for employee $G(B)$ on the effort of employee $B(G)$ is not necessarily monotonic:

$$\frac{\partial e_b}{\partial \omega_G} = \frac{\sigma \omega_B^2 (\omega_B - \omega_G)}{(\sigma \omega_B + \omega_G)^3} \geq 0 \iff \sigma \geq \frac{\omega_G}{\omega_B}$$

(18)

$$\frac{\partial e_g}{\partial \omega_B} = \frac{\sigma \omega_G^2 (\omega_G - \sigma \omega_B)}{(\sigma \omega_B + \omega_G)^3} \leq 0 \iff \sigma \geq \frac{\omega_G}{\omega_B}$$

The following proposition summarizes these indirect effects of the implicit value of winning on the effort of the other competitor under different handicapping policy, which is the key factor that drives our main results of optimal handicapping choice in the next section.

**Proposition 3**

1. Under a handicapping policy ($h > 1$) where $\omega_G < \omega_B < v$,

   \[
   \frac{\partial e_b}{\partial \omega_G} < 0 \quad \text{and} \quad \frac{\partial e_g}{\partial \omega_B} > 0, \quad \text{if employee abilities are very different (} \sigma \ll \frac{\omega_G}{\omega_B} \text{)},
   \]

   \[
   \frac{\partial e_b}{\partial \omega_G} > 0 \quad \text{and} \quad \frac{\partial e_g}{\partial \omega_B} < 0, \quad \text{otherwise (} \sigma \geq \frac{\omega_G}{\omega_B} \text{)}.
   \]

2. Under a reverse handicapping policy ($h \leq 1$) where $\omega_G > \omega_B > v$,

   \[
   \frac{\partial e_b}{\partial \omega_G} \leq 0 \quad \text{and} \quad \frac{\partial e_g}{\partial \omega_B} \geq 0.
   \]
Proof. See the Appendix. ■

To better understand the full effect of handicapping on efforts, Proposition 3 reveals the pervasiveness of $h$ through the implicit valuations.

A handicapping policy obviously reduces both players’ valuations, but in different amounts. When abilities are sufficiently different ($\sigma < \frac{\omega_G}{\omega_B}$), a decline in $\omega_B$ causes employee $G$ to lower his effort ($\frac{\partial e_g}{\partial \omega_B} > 0$) since he is playing an even weaker employee $B$. However, employee $B$ lowers his effort when the value to employee $G$ increases ($\frac{\partial e_b}{\partial \omega_G} < 0$). This results stems from the fact that the increased effort of stronger player due to the increase in $\omega_G$ makes the first period contest even more asymmetric such that weaker player (employee $B$) has very little chance of winning. In this case, the weaker player simply reduce its effort cost in the first period.

On the other hand, when employees’ abilities are similar ($\sigma \geq \frac{\omega_G}{\omega_B}$), a change in $h$ has the opposite effect. Employee $B$ surprisingly lowers his effort further when the value to employee $G$ declines. Likewise, when $\omega_B$ increases, employee $G$ increases his effort. In other words, the weaker employee has a fair chance of winning the contest when employees’ abilities are similar and thus, tries to match his rival in the contest. However, the stronger employee, $G$, now finds it optimal to reduce his effort cost in the first period and tries to benefit from the handicapping policy in the second period, which makes the contest even more asymmetric.

In addition, a reverse handicapping ($h < 1$) increases both players’ valuations for winning the first period contest. An increase in $\omega_B$ causes employee $G$ to raise his effort ($\frac{\partial e_g}{\partial \omega_B} > 0$) to match the competitor in the contest. However, employee $B$ lowers his effort when the value to employee $G$ increases ($\frac{\partial e_b}{\partial \omega_G} < 0$). This is very similar to the case of when employees are very different under a handicapping policy: the increased asymmetry in the first period due to the increase in $\omega_G$, employee $B$ has very little chance of winning. Hence, employee $B$ finds it optimal to reduce his effort cost even for all ability differences ($\frac{\partial e_b}{\partial \omega_G} \leq 0$ and $\frac{\partial e_g}{\partial \omega_B} > 0$).

This indirect effect of reduced valuation of winning the first period contest is the key factor that mitigates the first period effort losses from handicapping and explains how handicapping can increase total effort if the contest is asymmetric enough. Inasmuch as what these effects have on overall effort remains to be seen in the next section.

4.2 Handicapping versus Reverse Handicapping

In the previous section, we showed that reverse handicapping ($h < 1$) has the direct effect of increasing the valuation of winning the first period ($\omega_G, \omega_B$), leading to higher total efforts in
the first period, but it also has a separate and unique indirect effect on each employee’s effort, 
\((\frac{\partial \omega_B}{\partial \omega_G} < 0, \frac{\partial \omega_G}{\partial \omega_B} > 0)\). Furthermore, a reverse handicap may increase the ability gap in the second period, lowering total efforts in the second period. Because the manager wants to maximize total efforts over both periods, she should consider all three effects to assess whether a reverse handicap might achieve this goal.

Let \(e_1^{Total}\) and \(e_2^{Total}\) be the total efforts of the first and second period, respectively. Given a handicapping policy \(h\) and ability difference \(\sigma\), the total expected equilibrium effort for both periods is

\[
E\left[ e^{Total} (h, \sigma) \right] = E\left[ e_1^{Total} (h, \sigma) + e_2^{Total} (h, \sigma) \right] = e_{1,b} + e_{1,g} + \left[ s_B \cdot 2 \cdot e_{f}^{2} + (1 - s_B) \cdot 2 \cdot e_{t}^{2} \right].
\]

where 
\[
e_{1,b} (\omega_B, \omega_G, \sigma) = \frac{\sigma \omega_G}{(\omega_B + \sigma)^2}, \quad e_{1,g} (\omega_B, \omega_G, \sigma) = \frac{\sigma \omega_B}{(\sigma \omega_B + 1 + \sigma h)^2},
\]
\[
\omega_B (h, \sigma) = v + v \left[ \left( \frac{\sigma}{\sigma + h} \right)^2 - \left( \frac{\sigma h}{1 + \sigma h} \right)^2 \right],
\]
\[
\omega_G (h, \sigma) = v + v \left[ \left( \frac{1}{1 + \sigma h} \right)^2 - \left( \frac{h}{\sigma + h} \right)^2 \right],
\]
\[
e_f^{2} (h, \sigma) = \frac{v(\sigma h)}{(\sigma + h)^2}, \quad e_t^{2} (h, \sigma) = \frac{v(\sigma h)}{(1 + \sigma h)^2}, \quad \text{and } s_B = \frac{\sigma \omega_B}{\sigma \omega_B + \omega_G}.
\]

Here, the manager chooses the handicapping policy \(h\) that maximizes the expected total efforts for both periods. Recall that since employees’ valuations are identical in the second period, their equilibrium efforts in that period are also the same; if employee \(B\) wins in the first period (with probability \(s_B\)), the equilibrium effort is \(e_f^{2}\) for both employees, whereas if he loses in the first period (with probability \(1 - s_B\)), it is \(e_t^{2}\).

It is important to note that \(h\) affects employees’ implicit valuations in the first period through \(\omega_B\) and \(\omega_G\). Given a manager’s handicapping policy in the second period, employees choose their efforts in the first period, fully anticipating the consequence of their choices in the second period.

**Proposition 4** When employee abilities are very different (\(\sigma\) is sufficiently small), a handicap policy \((h > 1)\) maximizes the expected total effort. Otherwise, when employee abilities are similar (\(\sigma\) is sufficiently large), a reverse handicap policy \((h < 1)\) maximizes total effort.

**Proof.** See the Appendix.  ■
There is a fundamental trade-off between the effort levels across two periods when the manager employs a handicapping policy. On the one hand, a handicapping policy \( (h > 1) \) always increases effort in the second period because it levels the playing field and encourages the weaker player. Yet favoring the loser reduces the incentives of players to win the first-period contest, tempering the gains from the second period.

On the other hand, reverse handicapping \( (h < 1) \) always increases effort in the first period because the benefit of winning the first period becomes more pronounced in the form of an advantage in the second period. However, this reduces effort in the second period because the winner of the first contest would no longer need to work as hard in the second period, tempering the gains from the first period.

Overall, the manager must balance these trade-offs when choosing a handicapping strategy. When employees are very different in their abilities (i.e., small \( \sigma \)), the handicapping policy \( (h > 1) \) can intensify competition between employees in the second period, and this benefit of increased effort exceeds the loss of effort in the first period. Both employees reduce effort in the first period to take advantage of handicapping, but employees do not race to the bottom due to the strategic indirect effect of the weaker employee \( B \) \( (\frac{\partial e_B}{\partial \omega_G} < 0 \) from Proposition 3). The value of stronger employee \( G \) decreases significantly (i.e., \( \omega_G < \omega_B < v \) from Proposition 2), thus lowering his first period effort. This has an indirect effect on the weaker employee’s effort since he now has a higher opportunity to win the first period contest against a more restrained stronger employee and increases his effort, mitigating the loss in total first period effort to the manager. This only occurs when employees are very different in their abilities (i.e., small \( \sigma \)), and in this case, a handicapping policy \( (h > 1) \) maximizes expected total effort.

In contrast, when employee abilities are very similar (i.e., large \( \sigma \)), the cost in lost effort associated with the handicapping policy \( (h > 1) \) is greater. Again, the value for winning the first period contest decreases for both employees \( G \) and \( B \), reducing their efforts. Unlike the case when \( \sigma \) is small (or employees are very different), the indirect strategic effect of the reduced implicit value causes the weaker employee to further reduce his effort \( (\frac{\partial e_B}{\partial \omega_G} > 0 \) from Proposition 3) instigating a true “race to the bottom”. Note that this indirect effect is greater than the indirect strategic effect on employee \( G \) since \( \omega_G < \omega_B < v \) (i.e., \( \frac{\partial e_B}{\partial \omega_G} \) \( > \frac{\partial e_G}{\partial \omega_G} \)). Furthermore, since the abilities are similar, any handicap would only marginally improve effort in the second period. Therefore, when employee abilities are very similar, the cost associated with a handicapping policy \( (h > 1) \) exceeds the benefit from the handicapping policy and thus,
a handicapping policy cannot be an optimal policy.

A reverse handicapping policy, on the other hand, motivates employees to compete more intensely in the first period, outweighing the costs of lackluster performance in the second period. In other words, the potential incentive problem from favoring the winner is not severe when players are very similar in their abilities. Even with a small advantage to the winner, the loser still has sufficient incentive to exert effort in the second period contest. In anticipation of such effort, the stronger player still responds to the contest with sufficiently high effort level in the second period. Hence, when the ability difference is small (i.e., \( \sigma \) is sufficiently large), the cost of effort loss in the second period is more than compensated by the increased effort in the first period – a reverse handicapping policy \( (h < 1) \) maximizes the expected total effort.

In particular, we show that it is beneficial to reverse handicap \( (h^* = 1/3) \), even when employees are identical in ability (i.e., \( \sigma = 1 \)).

**Corollary 1** When employees are identical in ability, \( \sigma = 1 \), a reserve handicap policy, \( h = \frac{1}{3} \), maximizes total effort.

This is in stark contrast to the static case, where a handicapping policy \( (h > 1) \) is optimal for all ability differences. The reverse handicapping only arises from the need to balance the fundamental trade-offs between the levels of efforts across two periods in a dynamic setting. For symmetric abilities, \( \sigma = 1 \), the total expected effort is \( \frac{(3h+1)v}{(h+1)^2} \), which is maximized by a reverse handicap policy, \( h = \frac{1}{3} \).

We illustrate the relationship between the relative ability \( \sigma \) and the optimal handicapping policy \( h \) in Figure 2-(a) below, when \( v = 1 \). This clearly demonstrates that a reverse handicapping policy \( (h < 1) \) is beneficial to the manager’s attempt to raise effort when employees are similar in their abilities (in this particular example, when \( \sigma > 0.36 \)). A handicapping policy \( (h > 1) \) is beneficial only when employees are sufficiently different in their abilities (when \( \sigma < 0.36 \)).

Next, we investigate the relationship between the relative ability \( \sigma \) and the expected total effort from both periods \( E(e^{Total}) \) under the optimal handicapping policy \( h^* \). As Figure 2-(b) shows, the expected total effort \( E(e^{Total}) \) under the optimal policy \( h^* \) is always greater than the expected total effort without any handicapping policy (i.e., \( h = 1 \) for all \( \sigma \)).

To better understand the underlying forces behind this result, we decompose the total effort into individual effort by period, as shown in Figure 3. First, we note that there is a single
crossover between the first and second period effort at the point where the optimal handicapping policy is $h = 1$ (in this particular case, $\sigma = 0.36$). A handicapping policy of $h = 1$ is equivalent to the case of without any handicapping policy (or static contest case). Hence, effort levels from both the first and the second periods are identical. Second, Figure 3-(b) also illustrates that effort of each employee are equal in the second period where $v_G = v_B$ (Lemma 1).

When employees’ abilities are sufficiently different ($\sigma < 0.36$), a handicapping policy of $h > 1$ clearly raises the second period effort by both employees (Figure 3-(b)). Moreover, the first period effort by the weaker employee B is greater than that of the stronger employee G (Figure 3-(a)). This is because the potential effort loss in the first period is mitigated by the indirect effect identified in Proposition 3. The decreased value of winning the first period, or “loser’s bonus”, has a strategic effect on employee G to reduce his effort. This reduction in effort from a restrained employee G causes employee B to raise his effort in the first period. In other words, the first-period change in effort is ambiguous and small. This effect is offset by the overwhelming increase in effort in the second period. Even though the stronger employee has an incentive to hold back in the first period, the weaker employee is still more likely to lose and gain the handicap in the second period, consequently making the contest more competitive (Figure 3-(b)).

On the other hand, when employees’ abilities are quite similar ($\sigma > 0.36$) a reverse handicapping increases the first period effort by raising the value of winning in the first period for both employees (Figure 3-(a)). While the strategic effect from an increased valuation of winning
Figure 3: Effort by Period and Employee Type as a Function of Ability Differences for $h = h^*$ in the first period is greater for stronger employee $G$ (Proposition 2), the indirect effect from the weaker player is also increased effort. These effects are higher than the effort reduction from increased asymmetry in the second period contest.

5 Extensions

5.1 Commitment Failure

One of the main assumptions of the model is that the manager can credibly commit to a handicapping or reverse handicapping policy prior to the first period contest. This seems reasonable since firms tend to operate in an environment with enforceable contracts with its employees, clients, and suppliers. While this is an acceptable assumption, the likelihood of lacking a commitment device is equally plausible, especially in promotion settings, and deserves examination.\footnote{In practice, we find many other commitment problems such as when a referee might be tempted to favor the losing team in the second half of a sports contest, or a professor could want to reward a student who shows significant improvement on the final exam, to the detriment of a student who has performed well all along.}

In particular, when abilities are sufficiently similar, the manager might find it very tempting to switch from the reverse handicapping policy (which clearly increases the first period effort but lowers the second period effort) to a handicapping policy at the beginning of the second period in the absence of commitment. Employees would fully anticipate this, restraining their efforts
in the first period. The manager, knowing the employees are strategic, lessens the handicapping policy, and so on.

To accommodate this issue, we relax the commitment assumption by considering a case where the manager chooses a policy $h$ only after the first period. The employees still maximize their utility over both periods where first period efforts perfectly anticipate the optimal $h$ without commitment in the second period. Since the manager cannot commit to a policy, she only maximizes second period efforts based on the strategically adjusted equilibrium efforts in the first period. Hence, given effort in the first period, the manager maximizes

$$\max_h \left( s_B \cdot 2 \cdot e^f + (1 - s_B) \cdot 2 \cdot e^l \right).$$

(20)

**Proposition 5** Total effort from both employees under commitment is greater than effort with no commitment for all ability differences.

**Proof.** See the Appendix.

Since the manager only considers the effect of $h$ on the first period success function, total first period efforts are lower than with commitment. Even when the manager knows employees have an expectation of handicapping ($h > 0$), the manager still handicaps the loser of the first period, because handicapping is optimal for all ability differences (Proposition 1) and thus increases the second period effort levels.

Consistent with Freixas et al. (1985), who explored the consequences of non-commitment, Proposition 5 shows that without commitment, employees anticipate the manager will handicap the loser, and "hide" their types with less effort, resulting in a similar ratchet effect.

The importance of this proposition is further highlighted in the next section, where we compare our commitment model with the benchmark. In the benchmark, we consider the case where the manager has the perfect information about employees’ types and thus, there is no need for commitment. Even with imperfect information, handicapping with commitment dominates handicapping without commitment with perfect information.

### 5.2 Perfect Information

To illustrate the effectiveness of a dynamic nature of handicapping policy for eliciting employees’ efforts, we compare our main model with a benchmark scenario. In the two period contest setting, we assume the manager receives a *noisy* signal about who is the weaker employee by observing
only the outcome of the first period contest. In the benchmark case, we assume instead that the manager perfectly learns the abilities of both employees at the end of the first period through interaction with them during the first period contest.\footnote{In our setting, we use a contest as a broadly defined competition between employees and the manager identifies employee types based on overall evaluations over a certain period of time. During this period (first period contest), the manager often recieve signal about employee types other than their outcomes (which is also influenced by external factors such as luck). Hence, the manager can sometimes precisely identify who the weaker or stronger employee is after the initial evaluation period.} Hence, there is no uncertainty about the employee types in the second period. Might this invalidate the use of handicapping based on the first period result?

If the manager knows employee types perfectly, she can achieve the first-best outcome in the second period by applying a handicap, \( h = \frac{1}{\sigma} \) (Proposition 3), yielding \( \frac{v}{2} \) in total second period effort. Since employees fully anticipate the manager will perfectly identify their type in the second period there is no strategic component of effort in the first period. Furthermore, commitment is no longer a requirement. Thus, the first period in the repeated contests reverts to the static contest result where total effort is equal to \( \frac{2\sigma v}{(\sigma + 1)^2} \). With perfect information, the total effort from both players in both periods is

\[
\epsilon^{Total} = \frac{2\sigma v}{(\sigma + 1)^2} + \frac{v}{2} = \frac{v(1 + 6\sigma + \sigma^2)}{2(1 + \sigma)^2},
\]

where \( \epsilon^{Total} \) denotes the effort level under perfect information, which simplifies to \( v \) when \( \sigma = 1 \).

We can easily see \( \epsilon^{Total} < \epsilon^{Total} \) under a handicapping policy region (i.e., when employees are very different or \( \sigma \) is sufficiently low), since the second period effort is always higher under perfect information (\( \epsilon_2^{Total} < \epsilon_2^{Total} \)), and the first period effort is lower than in the static case under handicapping (\( \epsilon_1^{Total} < \epsilon_1^{Total} \) from Proposition 2). However, we find that even if the manager has perfect information about employee types, it is still optimal to commit to a reverse handicapping based on the first period contest result when employees are quite similar, or \( \sigma \) is sufficiently large.

**Proposition 6** There exists a \( \sigma^* \) such that for all \( \sigma \in [\sigma^*, 1] \), the total effort under a reverse handicapping policy based on the first period contest result is greater than a handicapping policy under perfect information: \( \epsilon^{Total} > \epsilon^{Total} \).

**Proof.** See the Appendix. \( \blacksquare \)
In spite of having perfect information about employee types, the manager can be better off by forgoing this information and still reverse handicapping the winner when employees are quite similar.\textsuperscript{14} In this case, the employees’ future consideration raises the first period effort due to the reverse handicapping. This increase in the first period effort can more than compensate the effort loss from the inefficiency caused by using noisy information based on the first period contest result (instead of using perfect information about employee types). For example, in the symmetric case of $\sigma = 1$, the optimal handicapping is $h^* = \frac{1}{3}$, and the total effort is $e^{Total} = \frac{9}{8}v > v = e^{Total}$.

Proposition 6 highlights the value of a dynamic nature of handicapping policy; by dividing the contest into separate, but mutually dependant contests, a manager can increase total effort by committing to a reverse handicapping policy above the level which simply perfect information can achieve. Therefore, the effect of handicapping or reverse handicapping policy is not merely shifting employee’s efforts between two periods, but in fact increases the total effort level above the perfect information case due to employees’ strategic behaviors arising from the dynamic relationship between two contests through handicapping policy.

6 Conclusion

Managers constantly face the problem of motivating a heterogeneous workforce. This issue aggravates the effectiveness of contests as a means to increase labor efforts. A typical response is simply to increase the incentive, but that option would generate, at best, additional effort proportional to the cost of the prize. We show that by dividing the contest into separate, but mutually dependent contests, a manager can increase total effort by committing to a handicapping or reverse handicapping policy. A conflict arises, however, between favoring a poor performer or rewarding a top performer. When the manager only has information on relative ability differences between employees, such that they are either sufficiently similar or sufficiently different, a two-period contest will serve the manager’s objectives. A natural concern is that favoring the loser of the first period increases effort in the second period at the expense of reducing each employee’s incentive to win the first period and the well-known “ratchet effect” would ensue. When employees’ abilities are sufficiently similar, this concern is justified: the reduction

\textsuperscript{14}Note that under a handicapping region ($h > 1$), it is always $e^{Total} < e^{Total}$. This is because the second period effort is always higher under perfect information ($e_2^{Total} < e_2^{Total}$) and the first period effort is lower than a static case under handicapping (Proposition 2); i.e., $e_1^{Total} < e_1^{Total}$.\textsuperscript{27}
in effort in the first period more than offsets the higher expected effort in the second period. Therefore, the standard handicapping strategy of favoring the loser of the first period would lower, rather than raise, total effort. The effort-maximizing contest design instead requires a reverse handicapping policy of favoring the winner.

These results complement the findings of Meyer (1991, 1992), who uses a Lazear-Rosen (1981) difference-form contest to model multiperiod competition between two equally capable employees for a job promotion. Our results entail a Tullock (1980) ratio-form contest between two asymmetric employees. We identify the optimal handicapping policy for a full range of ability differences and thus provide managers with guidelines for favoring winners or losers when faced with repeated contests environments.

When the relative ability gap between weaker and stronger employees is sufficiently large, we find support for the standard handicapping policy of favoring the loser of the first period. Even though the second-period handicapping decreases the total first period effort, the employees' strategic consideration for the second-period handicap mitigates this effort loss in the first period. This is because the second period handicapping hurts the stronger employee's first-period incentives more than it hurts the weak employees' first-period incentives. Hence, the first-period contest becomes more equitable through the second-period handicap. This mitigates the negative impact on total first-period effort for very large ability differences.

On the other hand, when employees' abilities are sufficiently similar, we find that a reverse handicapping policy of favoring the winner of the first period contest is optimal. With little difference in their abilities, the extent of advantage the winner of the first contest receives is also small. Accordingly, the loser of the first period contest still has a good chance to win in the second period even with a small disadvantage, and thus experiences sufficient incentive to exert effort. Hence, the increase in effort in the first period more than offsets the effort loss in the second period when employees' abilities are sufficiently similar.

Also, we show that a manager can increase total effort by committing to a reverse handicapping policy above the level which simply perfect information can achieve. Therefore, the effect of handicapping or reverse handicapping policy is not merely shifting employee's efforts between two periods, but in fact increases the total effort level above the perfect information case due to employees' strategic behaviors arising from the dynamic relationship between two contests through handicapping policy. Furthermore, this implies that even if the manager knew the ability of each employee, a reverse handicapping policy has a greater incentive effect than
evening the contest with a handicapping policy. This result uniquely contributes to existing contest literature concerning maximizing effort and handicapping.

While maximizing effort is a common objective, in practice a manager might have several alternative objectives, such as accurately identifying the good employee in order to make promotion, transfer, or termination decisions. We can apply the current model to examine this important issue (see the Technical Appendix for the detailed analysis). If the goal of the manager is to identify the more qualified employee, then she should always reverse handicap regardless of the ability differences. By reverse handicapping, the better employee reveals their type by increasing their probability of winning through increased effort in the first period. In contrast, handicapping induces both employees to “hide” their type by not exerting as much effort in the first period in order to receive the handicap in the second period. This further increases the uncertainty in identifying the better employee. When choosing the winner of either the first period or second period contest as the standard for promotion, the probability of correctly identifying the better employee increases when a reverse handicapping policy is used.¹⁵

Finally, there are many other factors that affect the effectiveness of a handicapping policy such as fairness or prize structure. For instance, handicapping the weaker employee may cause the stronger employee to feel that the contest has been unfairly altered such that their work appears under-appreciated, which may eventually lead to disenchantment for the stronger player. This point is worth examining from a goal theory perspective, as do Murphy et al. (2004). Our paper complements this goal theory perspective and contributes to understanding why favoring the loser could lower the effort from a stronger employee. Yet the proposed model sometimes seems to favor the Matthew Effect, by which winners only win because they have won in the past, and not due to their superior ability (Merton 1968). We believe that our model can offer another explanation for the Matthew Effect. Also, we only examine the two-employee case whereas other research has examined a much larger pool of participants, in which case the optimal proportion of winners in a contest can be an important issue (Lim 2010). However, if managers only have to distinguish between a pair of leaders, it might not be practical to include more than two agents in a competition, say for the same client account. Nevertheless, an application to the

¹⁵In the Technical Appendix, we examine the extent to which the handicapping model can improve accuracy of a contest. We explicitly consider two possible rules in choosing the better employee in the contest. The first rule is to choose the winner of the first round while the second rule is to choose the winner of the second round. Under both rules, we show that a reverse handicapping is always optimal for all $\sigma \leq 1$. 
multi-employee case would broaden the implications of our study.
Appendix

Proof of Proposition 1:

For a given $\sigma \in [0, 1]$ and $p \in [0, \frac{1}{2})$, the manager solves the following maximization problem:

$$\max_{h>0} E(e^{Total}) = p \left( 2v \left( \frac{\sigma/h}{(1 + \sigma/h)^2} \right) \right) + (1 - p) \left( 2v \left( \frac{\sigma h}{(1 + \sigma h)^2} \right) \right).$$

The first-order condition with respect to $h$ yields:

$$\frac{\partial E(e^{Total})}{\partial h} = 2\sigma v \left( \frac{(1-p)(1-h\sigma)}{(1 + \sigma h)^3} - \frac{p(h - \sigma)}{(h + \sigma)^3} \right).$$

It can be easily seen that $\left[ \frac{\partial E(e^{Total})}{\partial h} \right]_{h=1} = 2\sigma v \left( \frac{(1-2p)(1-\sigma)}{(1+\sigma)^4} \right) \geq 0$ (equality holds when $\sigma = 1$), and $\frac{\partial E(e^{Total})}{\partial h} < 0$ for all $h > \frac{1}{\sigma} (> 1)$. By continuity, there must be at least one point $h^* \in [1, \frac{1}{\sigma})$ at which $\left[ \frac{\partial E(e^{Total})}{\partial h} \right]_{h=h^*} = 0$.

Moreover, we can show that $\frac{\partial E(e^{Total})}{\partial h} > 0$ for all $h < 1$.

1. When $h \leq \sigma(< 1)$, it is trivially satisfied.

2. When $\sigma < h < 1$, we show that $\frac{(1-p)(1-h\sigma)}{(1 + \sigma h)^3} > \frac{p(h - \sigma)}{(h + \sigma)^3}$ by rearranging the inequality

$$\frac{(1-p)(1-h\sigma)}{(1 + \sigma h)^3} > \frac{p(h - \sigma)}{(h + \sigma)^3} \iff \frac{(1-p)}{p} \left( \frac{h + \sigma}{1 + \sigma h} \right)^3 \left( \frac{1-h\sigma}{h - \sigma} \right) > 1$$

$$\iff p < \frac{(h + \sigma)^3(1-h\sigma)}{(1-\sigma^2)[h(1+h^2) + h(1+h^2)\sigma^2 - (1-6h^2 + h^4)\sigma]} = \overline{p}.$$

In the second inequality, as $p \downarrow 0$, the (LHS) goes to infinity, and thus, the inequality is always satisfied. More precisely, we can find the condition such that when $p < \overline{p}$, $\frac{\partial E(e^{Total})}{\partial h} > 0$ for all $h < 1$.

In summary, when $p < \overline{p}$, $\frac{\partial E(e^{Total})}{\partial h} > 0$ for all $h \leq 1$, and $\frac{\partial E(e^{Total})}{\partial h} < 0$ for all $h \geq \frac{1}{\sigma} (> 1)$. Therefore, $E(e^{Total})$ attains its maximum at some point $h^* \in [1, \frac{1}{\sigma})$ where $\left[ \frac{\partial E(e^{Total})}{\partial h} \right]_{h=h^*} = 0$ by the mean value theorem. Hence, the optimal handicapping policy is strictly smaller than $\frac{1}{\sigma}$, which is the case under perfect information. \(\square\)

Proof of Proposition 2:
First, it is obvious that \( \omega_G = \omega_B = v \) when \( h = 1 \). Next, note that \( \omega_B = v + \left[ \frac{\sigma^2 v}{(\sigma + h)^2} - \frac{(\sigma h)^2 v}{(1 + \sigma h)^2} \right] \), \( \omega_G = v + \left[ \left( \frac{1}{1 + \sigma h} \right)^2 v - \left( \frac{h}{\sigma + h} \right)^2 v \right] \). Hence,

\[
\omega_G - \omega_B = \left[ \left( \frac{1}{1 + \sigma h} \right)^2 - \left( \frac{h}{\sigma + h} \right)^2 \right] v - \frac{\sigma^2}{(\sigma + h)^2} + \frac{(\sigma h)^2}{(1 + \sigma h)^2} v \tag{22}
\]

Therefore, \( \omega_G - \omega_B > 0 \) if and only if \( h < 1 \) (note that \( \sigma < 1 \)).

Moreover, \( \omega_B = v + \left[ \frac{\sigma^2 v}{(\sigma + h)^2} - \frac{(\sigma h)^2 v}{(1 + \sigma h)^2} \right] > v \) if and only if \( h < 1 \), because \( \frac{\sigma^2 v}{(\sigma + h)^2} < \frac{(\sigma h)^2 v}{(1 + \sigma h)^2} \iff 1 < h \). The results in the proposition follow. \( \square \)

**Proof of Proposition 3:**

From the first order condition, \( \frac{\partial e_b}{\partial \omega_G} = \frac{\sigma \omega_B^2 (\sigma \omega_B - \omega_G)}{(\sigma \omega_B + \omega_G)^3} \) and \( \frac{\partial e_a}{\partial \omega_B} = -\frac{\sigma \omega_B^2 (\sigma \omega_B - \omega_G)}{(\sigma \omega_B + \omega_G)^3} \).

1. Under a handicapping policy, \( \omega_G < \omega_B < v \). Hence, if \( \sigma < \frac{\omega_G}{\omega_B} \), then \( \frac{\partial e_b}{\partial \omega_G} = \frac{\sigma \omega_B^2 (\sigma \omega_B - \omega_G)}{(\sigma \omega_B + \omega_G)^3} < 0 \), \( \frac{\partial e_a}{\partial \omega_B} > 0 \), \( \frac{\partial e_a}{\partial \omega_G} > 0 \). Otherwise (\( \sigma \geq \frac{\omega_G}{\omega_B} \)), \( \frac{\partial e_a}{\partial \omega_B} > 0 \), \( \frac{\partial e_a}{\partial \omega_G} < 0 \).

2. Under a reverse handicapping policy, \( \omega_G > \omega_B > v \). Thus, \( \frac{\partial e_b}{\partial \omega_G} = \frac{\sigma \omega_B^2 (\sigma \omega_B - \omega_G)}{(\sigma \omega_B + \omega_G)^3} < 0 \) and \( \frac{\partial e_a}{\partial \omega_G} = \frac{\sigma \omega_B^2 (\omega_G - \sigma \omega_B)}{(\sigma \omega_B + \omega_G)^3} > 0 \).

3. Without a handicapping policy, \( \omega_G = \omega_B = v \). Thus, \( \frac{\partial e_b}{\partial \omega_G} = 0 \) and \( \frac{\partial e_a}{\partial \omega_B} = 0 \). \( \square \)

**Proof of Proposition 4:**

First, we show that there exists a unique equilibrium of efforts by both employees for all \( \sigma \) and \( h \). We have already established that the first- and second-order conditions are satisfied, yielding existence of an equilibrium. Therefore, to prove uniqueness, we examine the Hessian of \( U_B \) and \( U_G \),

\[
H = \begin{bmatrix}
\frac{\partial^2 U_B}{\partial e_b^2} & \frac{\partial^2 U_B}{\partial e_b \partial e_g} \\
\frac{\partial^2 U_B}{\partial e_g \partial e_b} & \frac{\partial^2 U_B}{\partial e_g^2} \\
\frac{\partial^2 U_G}{\partial e_b^2} & \frac{\partial^2 U_G}{\partial e_b \partial e_g} \\
\frac{\partial^2 U_G}{\partial e_g \partial e_b} & \frac{\partial^2 U_G}{\partial e_g^2}
\end{bmatrix},
\tag{23}
\]

which simplifies to

\[
H = \begin{bmatrix}
\frac{\partial^2 s_B}{\partial e_b^2} v_B & -\frac{\partial^2 s_B}{\partial e_b \partial e_g} v_B \\
-\frac{\partial^2 s_B}{\partial e_g \partial e_b} v_B & \frac{\partial^2 s_B}{\partial e_g^2} v_B
\end{bmatrix},
\tag{24}
\]

32
since valuations are exogenous to efforts. If $H$ is negative definite for all $e_b$ and $e_g$, then the equilibrium is unique (Rosen 1965). This claim is easy to verify since the first component is negative by assumption and the determinant

$$|H| = \frac{\partial^2 s_B}{\partial e^2_b} \frac{\partial^2 s_B}{\partial e^2_g} v_B v_G - \frac{\partial^2 s_B}{\partial e_b \partial g} \frac{\partial^2 s_B}{\partial e_b \partial e_g} v_B v_G > 0,$$

is positive definite, provided that

$$-\frac{\partial^2 s_B}{\partial e^2_b} \frac{\partial^2 s_B}{\partial e^2_g} > \frac{\partial^2 s_B}{\partial e_b \partial g} \frac{\partial^2 s_B}{\partial e_b \partial e_g}. $$

(26)

We have already established existence, so we can evaluate Equation (26) for each success function:

From Equation (10), when $\Pr (B \text{ wins second period} \mid B \text{ loses first period}) = s_{B|L} = \frac{\sigma h e_b}{(\sigma e_b + e_g)^2}$, $|H| > 0$, because $\frac{h^2 \sigma^2}{(e_g + e_b h \sigma)^2} > 0$.

From Equation (11), when $\Pr (B \text{ wins second period} \mid B \text{ wins first period}) = s_{B|W} = \frac{e_b (\frac{\pi}{h})}{(e_b (\frac{\pi}{h}) + e_g)}$, $|H| > 0$, because $\frac{h^2 \sigma^2}{(e_b \sigma + e_g)^2} > 0$.

From Equation (1), when $\Pr (B \text{ wins first period}) = s_B = \frac{\sigma e_b}{(\sigma e_b + e_g)}$, $|H| > 0$, because $\frac{\sigma^2}{(e_g + e_b \sigma)^2} > 0$.

Next, we show that the optimal handicapping policy is decreasing in $\sigma$. From Equation (19), we know that for a given $\sigma$, total effort is

$$E \left[ e^{Total} (h, \sigma) \right] = e_{1b} + e_{1g} + \left[ s_B \cdot 2 \cdot e_2^f + (1 - s_B) \cdot 2 \cdot e_2^f \right],$$

(27)

where

$$e_{1b} (\omega_B, \omega_G, \sigma) = \frac{\sigma \omega_G}{\omega_B + \sigma}, e_{1g} (\omega_B, \omega_G, \sigma) = \frac{\sigma \omega_B}{\omega_G + \sigma}, s_B = \frac{\sigma \omega_B}{\sigma \omega_B + \omega_G},$$

$$\omega_B (h, \sigma) = v + v \left[ \frac{\sigma}{\sigma + h} \right] - \left[ \frac{\sigma h}{1 + \sigma h} \right]^2, \omega_G (h, \sigma) = v + v \left[ \frac{1}{1 + \sigma h} \right] - \left( \frac{h}{\sigma + h} \right)^2,$$

$$e_2^f (h, \sigma) = \frac{v (\sigma h)}{(\sigma + h)^2}, \text{ and } e_2' (h, \sigma) = \frac{v (\sigma h)}{(1 + \sigma h)^2}.$$

At equilibrium $h = h^*$, the first-order condition with respect to $h$ satisfies

$$\frac{\partial E \left[ e^{Total} \right]}{\partial h} \bigg|_{h=h^*} = \frac{\partial E \left[ e^{Total} \right]}{\partial \omega_B} \frac{d \omega_B}{dh} + \frac{\partial E \left[ e^{Total} \right]}{\partial \omega_G} \frac{d \omega_G}{dh} + \frac{d E \left[ e^{Total} \right]}{dh}$$

$$= \left[ \frac{\partial e_{1b}}{\partial \omega_B} + \frac{\partial e_{1g}}{\partial \omega_B} + 2 \left( e_2^f - e_2' \right) \frac{\partial s_B}{\partial \omega_B} \right] \frac{d \omega_B}{dh} + \left[ \frac{\partial e_{1b}}{\partial \omega_G} + \frac{\partial e_{1g}}{\partial \omega_G} + 2 \left( e_2^f - e_2' \right) \frac{\partial s_B}{\partial \omega_G} \right] \frac{d \omega_G}{dh}$$

$$+ 2 s_B \frac{d e_2^f}{dh} + 2 (1 - s_B) \frac{d e_2'}{dh} = 0.$$
where \( \frac{\partial e_{1,b}}{\partial \omega_B} = \frac{2\sigma_B \omega_B^2}{(\sigma_B + \omega_B)^3} > 0 \), \( \frac{\partial e_{1,b}}{\partial \omega_B} = \frac{\sigma_G^2(\omega_G - \sigma_B)}{(\sigma_B + \omega_B)^3} > 0 \), \( e_f^t - e_f^t \geq 0 \) (when \( (1 - h^2)(1 - \sigma^2) \geq 0 \)),
\( \frac{\partial s_B}{\partial \omega_G} = \frac{\sigma_G}{(\sigma_B + \omega_B)^2} > 0 \), \( \frac{\partial s_B}{\partial \omega_G} = -2\sigma^2 \left( \frac{1}{(h + \sigma)^3} + \frac{h}{(1 + \sigma h)^3} \right) < 0 \),
\( \frac{\partial \omega_G}{\partial h} = -2\sigma \left( \frac{h}{(h + \sigma)^3} + \frac{1}{(1 + \sigma h)^3} \right) < 0 \),
\( \frac{\partial e_{1,b}}{\partial \omega_G} = \frac{2\sigma_G^2 \omega_G^2}{(\sigma_B + \omega_B)^3} > 0 \), \( \frac{\partial s_B}{\partial \omega_G} = \frac{-\sigma_B^2}{(\sigma_B + \omega_B)^2} \left( \frac{1 - \sigma^2(1 - h^2)}{\sigma_B - \omega_B} + 2\sigma_B \omega_B^2 \omega_G \right) \left( \frac{(1 + \sigma h)^3 + h (h + \sigma)^3}{(h + \sigma)^3(1 + \sigma h)^3} \right)
\) (28) simplifies to
\( \frac{\partial E}{\partial h} e_{Total} = 0 \rightleftharpoons \left[ \begin{array}{c} (\sigma_X^2 - \omega_B) (X^2Y^2 - h^2Y^2 + X^2 - \sigma X^2Y^2 - \sigma(1 - h^2)(2 - \sigma)(1 + 2h\sigma + h^2)) \\ \sigma(1 - h^2)(1 - \sigma^2) (X^2Y^2 - h^2Y^2 + X^2 - \sigma X^2Y^2 - \sigma(1 - h^2)(2 - \sigma)(1 + 2h\sigma + h^2)) \\ \sigma X^2Y^2 - h^2Y^2 + X^2 - \sigma X^2Y^2 - \sigma(1 - h^2)(2 - \sigma)(1 + 2h\sigma + h^2)) \\ \end{array} \right] \times (Y^3 + hX^3)
\)
\( \left[ \begin{array}{c} \sigma X^2Y^2 - h^2Y^2 + X^2 - \sigma X^2Y^2 - \sigma(1 - h^2)(2 - \sigma)(1 + 2h\sigma + h^2)) \\ (X^2Y^2 - h^2Y^2 + Y^2) \left( 2\sigma X^2Y^2 - h^2Y^2 + X^2 + (X^2Y^2 - \sigma^2(1 - h^2)(2 - \sigma)(1 + 2h\sigma + h^2)) \right) \\ \sigma X^2Y^2 - h^2Y^2 + X^2 - \sigma X^2Y^2 - \sigma(1 - h^2)(2 - \sigma)(1 + 2h\sigma + h^2)) \\ \end{array} \right] \times (hY^3 + X^3) + X^3Y^2 - Y^2 = 0
\)
where \( X = h + \sigma \) and \( Y = 1 + \sigma h \).
We differentiate $F(h, \sigma)$ with respect to $h, \sigma$. After a few algebraic steps, we can see that $\frac{dF}{dh} < 0$ for all $h > 0$ and $\sigma \in [0, 1]$, and $\frac{dF}{d\sigma} > 0$ if $h > 0$ and $\sigma \leq \hat{\sigma}$, where $\hat{\sigma}$ solves the equation $\frac{dF}{d\sigma}(h, \hat{\sigma}) = 0$. By the implicit function theorem, $\frac{dh}{d\sigma} = -\left(\frac{\frac{dF}{dh}}{\frac{dF}{d\sigma}}\right) < 0$. All we need to show is that when $\sigma$ is large such that $\sigma > \hat{\sigma}$, it is the case that $h^* < 1$. We prove it by contradiction. Suppose that when $\sigma$ is sufficiently large and the optimal handicapping policy is $h^* \geq 1$. Then, it must be the case that $
abla E\left[ e_{Total}(h, \sigma) \right]_{h=1} > 0$. However, we can easily see that

$$\frac{\partial E\left[ e_{Total}(h=1) \right]}{\partial h} = 2\left(1 + 5\sigma - 5\sigma^3 - 33\sigma^3 - 37\sigma^4 - 9\sigma^5 + 5\sigma^6 + \sigma^7\right) < 0,$$

if $\sigma > \hat{\sigma} \approx 0.38$

This contradicts the assumption. Thus, when $\sigma > \hat{\sigma}$, it is the case that $h^* < 1$.

In particular, when $\sigma = 1$, $E\left[ e_{Total}(h, \sigma = 1) \right] = v \frac{(1+3h)}{(1+h)^2} > 1$ from Equation (19). Total effort is therefore maximized at $h = 1/3$. $\square$

**Proof of Proposition 5:**

We find that $\left(\frac{\partial}{\partial h} e_{Total}\right)_{h=1} > 0$ for all $\sigma$. First, for sufficiently symmetric players, a reverse handicap maximizes the total effort in equilibrium, so handicapping the loser in the second period yields less effort. For sufficiently asymmetric players, however, a handicapping policy is used regardless of commitment. Let $\hat{h}$ be the optimal handicap without commitment. We know that $h > 1$ always decreases first period valuations and thus, total first period efforts. However, $\hat{h}$ only considers the effect on the first period success function, not on total efforts. Since $\hat{h} \neq h^*$ for all $\sigma$, $e_{Total}(h = h^*) > e_{Total}\left( h = \hat{h} \right)$. $\square$

**Proof of Proposition 6:**

We note that (1) $e_{Total} < \bar{e}_{Total}$ under a handicapping region (i.e., $\sigma$ is sufficiently small). Also, (2) when $\sigma = 1$, $e_{Total} = \frac{\bar{\sigma}}{2}v > \bar{e}_{Total} = v$. Hence, if $\left[ e_{Total}\right]$ is monotonically increasing in $\sigma$, there must exist a $\sigma^*$ such that for all $\sigma \in [\sigma^*, 1]$, $e_{Total} > \bar{e}_{Total}$ from continuity. Therefore, all we need to show is that $\left[ e_{Total}\right]$ is monotonically increasing in $\sigma$ under a reverse handicapping region (where $\omega_G > \omega_B$). Let $\sigma'$ be the cutoff such that when $\sigma > \sigma'$, a reverse handicapping is optimal.

First, it is easy to see that when $\sigma > \sigma'$,

$$\frac{\partial e_{Total}}{\partial \sigma} = \frac{\omega_B \omega_G (\omega_G - \sigma \omega_B)(\omega_G + \omega_B)}{(\sigma \omega_B + \omega_G)^3} > 0,$$

(30)
since $\omega_G > \omega_B$ under a reverse handicapping policy. Also,

$$\frac{\partial e_2^{Total}}{\partial \sigma} = 2h \left[ \frac{(1-\sigma h)\omega_2^2 + 2h\sigma(1-\sigma^2)(1+\sigma(3h-h^3+\sigma))\omega_B\omega_G - (\sigma-h)\sigma^2\omega_2^3}{(\sigma\omega_B + \omega_G)^2} \right]$$

(31)

In particular,

$$(1-\sigma h)\omega_2^2 (h + \sigma)^3 + 2h\sigma(1-\sigma^2)(1+\sigma(3h-h^3+\sigma))\omega_B\omega_G - (\sigma-h)\sigma^2\omega_2^3 (1 + \sigma h)^3

\geq \left[(1-\sigma h) (h + \sigma)^3 + 2h\sigma (1-\sigma^2)(1+\sigma(3h-h^3+\sigma)) - (\sigma-h)\sigma^2 (1 + \sigma h)^3 \right] \omega_B^2

$$

Let $F(\sigma) = (1-\sigma h) (h + \sigma)^3 + 2h\sigma (1-\sigma^2)(1+\sigma(3h-h^3+\sigma)) - (\sigma-h)\sigma^2 (1 + \sigma h)^3$. Then,

$$\frac{\partial F(\sigma)}{\partial \sigma} = h \left[ 8\sigma(1-3\sigma^2) - 6h^2\sigma(1-\sigma^2)^2 - h(3-h^2)(5\sigma^4+6\sigma^2-3) \right].$$

We can see that when $\sigma > \hat{\sigma} = \sqrt{\frac{2\sqrt{5} - 3}{5}} \approx 0.616$, $(5\sigma^4+6\sigma^2-3) > 0$ and $1-3\sigma^2 < 0$, thus $\frac{\partial F(\sigma)}{\partial \sigma} < 0$. This implies that $F(\sigma) \geq F(1) = 0$ for all $\sigma \in [\hat{\sigma}, 1]$.

Therefore, we show that when $\sigma$ is sufficiently large ($\sigma > \hat{\sigma}$),

$$\frac{\partial e_2^{Total}}{\partial \sigma} = 2h \left[ \frac{(1-\sigma h)\omega_2^2 (h + \sigma)^3 + 2h\sigma(1-\sigma^2)(1+\sigma(3h-h^3+\sigma))\omega_B\omega_G - (\sigma-h)\sigma^2\omega_2^3 (1 + \sigma h)^3}{(\sigma\omega_B + \omega_G)^2} \right] > 0$$

(32)

From (30) and (32), $[e^{Total}]$ is monotonically increasing in $\sigma$ ($\frac{\partial e_2^{Total}}{\partial \sigma} = \frac{\partial e_1^{Total}}{\partial \sigma} + \frac{\partial e_2^{Total}}{\partial \sigma} > 0$).

Hence, when $\sigma$ is sufficiently large where a reverse-handicapping is optimal, there must exist a $\sigma^*$ such that for all $\sigma \in [\sigma^*, 1]$, $e^{Total} > \bar{e}^{Total}$. □
References


